Saving Schrödinger's Cat: It's About Time (not Measurement)

Thomas Pashby*

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In this paper I argue for a novel resolution of Schrödinger's cat paradox by paying particular attention to the role of time and tense in setting up the problem. The quantum system at the heart of the paradoxical situation is an unstable atom, primed for indeterministic decay at some unknown time. The conventional account gives probabilities for the result of instantaneous measurements and leads to the unacceptable conclusion that the cat can neither be considered alive nor dead until the moment the box is opened (at a time of the experimenter's choosing). To resolve the paradox I reject the status of the instantaneous quantum state as 'truthmaker' and show how a quantum description of the situation can be given instead in terms of time-dependent chance propositions concerning the time of decay, without reference to measurement.

Aside from its arresting imagery, the cat paradox has enjoyed a lasting role in the foundations of quantum theory because, as was Schrödinger's intention, it serves as a microcosm of wider difficulties with the role of measurement in the foundations of quantum mechanics. In this sense, it provides a testbed for resolutions of the now infamous Measurement Problem. A novel resolution of the paradox, therefore, may lead to a new perspective on measurement and a distinctive interpretation of the theory.

One can think of the task of interpreting of quantum mechanics as that of providing a semantics for the quantum state. I take the quantum state at issue in Schrödinger's cat paradox to correspond to an ensemble of histories (i.e., a collection of models of tense logic) which vary as to the time of decay. That is, instead of being determined by a choice of the experimenter, the time of decay displays a probabilistic dependence on the quantum state in much the same way as does the position in space of an outcome in other characteristic quantum experiments.

The conclusions reached in the case of Schrödinger's cat may be generalized throughout quantum mechanics with the means of event time observables (interpreted as conditional probabilities), which play the role of the time of decay for an arbitrary system. This allows for an interpretation of the theory in which the time of measurement plays no role, and provides the means to resolve another well-known quantum paradox—Wigner's friend paradox—along similar lines. Furthermore,

*Provost's Postdoctoral Scholar, School of Philosophy, University of Southern California; pashby@usc.edu. *This is a draft—please do not cite.*

I show how the novel characterization of *temporal* indeterminism that results follows from fundamental principles of quantum theory, and cannot easily be replicated or replaced by competing interpretations.

This distinctive way of thinking about quantum mechanics, which I call the Stochastic Histories interpretation, proceeds by providing an alternative semantics for the theory while leaving the mathematical formalism alone. Since on this interpretation the observer plays no role in determining the time of an experimental outcome, it will appeal to philosophers seeking a pristine interpretation of the theory who, like David Lewis, are not ready to take lessons from quantum mechanics until "it is purified of supernatural tales about the power of the observant mind to make things jump" (Lewis, 1986b, p. xi). I contend that the idea that the observer may choose the time at which an outcome obtains lies at the root of Schrödinger's cat paradox, and this idea is one that we can—and must—avoid.

1. Schrödinger's Cat Paradox

The standard characterization of a paradox is given succinctly by Sainsbury (2009) as "an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises" (p. 1). Resolving a paradox, then, requires finding a way to palatably bite the bullet, or rejecting one or more premises, or revealing a flaw in the reasoning. Schrödinger's cat fits this model fairly well and, finding the conclusion less than appetizing, I will be adopting a combination of those two strategies in order to avoid it. Before stating the premises, let us remind ourselves of how (and why) Schrödinger sets up this situation:

> A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, *so* small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The ψ-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.
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> It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be *resolved* by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks. (Trimmer, 1980, p. 328)

I will confine myself to two immediate remarks. First, it is clear that Schrödinger regards the common sense description of the situation as maintaining the cat is alive if the radioactive sample has not decayed, while if the radioactive sample has decayed then it is dead. These conditional statements are *mixed* tense: the present tense state of the cat is said to depend on what happened to the sample in the past. Second, Schrödinger's final remark indicates his intention for introducing the example: it is supposed to make us leery of accepting the idea that the probabilistic quantum state provides a literal representation of the situation in the chamber. That is, since observation of the system reveals a determinate state of affairs, the relationship of the "blurred" quantum state to the reality of the situation must be more like that of an out of focus photograph to the unblurred state of affairs it represents, rather than the faithful representation of a blurry situation offered by an in focus photograph of fog.

These are two themes I will take up in my resolution of the paradox, but first it will be instructive to provide a logical reconstruction. Following van Fraassen (among others), I take the problematic aspect of the case to be as follows:

> This example is disturbing, because when it comes to cats (and other macroscopic objects) we feel sure that whatever we see when we open the box was already there. Or else, the other thing we might have seen was there; it is not inconceivable that the cat was alive till we opened the box, and died at that very moment. But our sensibilities are miffed, if not outraged, by the idea that it was not true that the cat was dead and also not true that it was alive. (van Fraassen, 1991, p. 263)

That is, the paradoxical conclusion to which we are led is that it is neither true that the cat is dead nor true that the cat is alive until the instant the box is opened.

This conclusion follows from the following assumptions made by the orthodox account of quantum mechanics:[1]

State A physical system has a unique quantum mechanical state providing a complete description of the system.

Born The state provides probabilities for outcomes of measurements according to the Born Rule.

Prop A proposition φ corresponds to:

(a) a projection P_φ

(b) a 'yes/no' instantaneous measurement

(c) a property of the system

Truth A proposition about a system is true at a time t if and only if a measurement at time t is certain to reveal the corresponding outcome.

Measure Outcomes of a measurement are:

(a) mutually exclusive and exhaustive

[1] By this I mean the sort of account of the theory that you would find in a good physics textbook. It is not clear to what extent it qualifies as an 'interpretation' of the theory.

(b) determinate and repeatable

These are general principles of the orthodox account of quantum mechanics. They are indeed contentious, but the reason for the discontent is that together they lead to unpalatable consequences, not that they are in themselves implausible. For now, let us just see where they take us.

One of the interesting features of this case is that it does not require a detailed specification of the dynamics of the system: an atomic decay process is described by an exponential decay law and so we need only the half-life of the substance to determine the probability that an atom has decayed at some time t.[2] The half-life is an hour, says Schrödinger. There are two outcomes of an instantaneous measurement here: either the atom has not yet decayed, or it has decayed. To these outcomes correspond two propositions:

P1 The atom has not decayed.

P2 The atom has decayed.

According to the orthodox account, the quantum mechanical state provides the probabilities for those propositions being found to be true on measurement, and the propositions are true only if the corresponding outcome is certain to be found on measurement.

The paradoxical conclusion regarding the indeterminate state of the cat can be reached in just a few steps:

1. The truth at time t of a proposition about the system depends only on the instantaneous state of the system, $|\psi_t\rangle$. (**State**; **Prop**)

2. The state of the system at time t is
$|\psi_t\rangle = \alpha_t|undecayed\rangle + \beta_t|decayed\rangle$, with $|\alpha_t|^2 + |\beta_t|^2 = 1$.

3. At all times $t > 0$, the probability that the atom has not decayed is
$\Pr(\text{undecayed at } t) = |\alpha_t|^2 < 1$. (**Born**)

4. At all times $t > 0$, the probability that the atom has decayed is
$\Pr(\text{decayed at } t) = |\beta_t|^2 < 1$. (**Born**)

5. 'The atom has not decayed' (P1) is true at t *iff* $\Pr(\text{undecayed at } t) = 1$; 'The atom has decayed' (P2) is true at t *iff* $\Pr(\text{decayed at } t) = 1$ (**Truth**).

6. $\therefore$ At all times $t > 0$, it is not the case that the atom has decayed, neither is it the case that the atom has not decayed. (3., 4., 5.)

7. The cat is alive *iff* the atom has not decayed; the cat is dead *iff* the atom has decayed.

8. $\therefore$ At all times $t > 0$, the cat is neither dead nor alive. (6., 7.)

[2]From hereon in I assume we are dealing with a single atom so that a half-life determines a well-formed probability.

The derivation relies on the idea that the state of the system is given as a *superposition* (or sum) of mutually exclusive alternatives, with the probability of an outcome being determined by the corresponding (complex) coefficient, which attaches a 'weight' to each term in the superposition. These weights change with time, which means that the probabilities for finding the atom decayed (and the cat dead) or the atom undecayed (and the cat alive) change with time. However, in general neither of these probabilities equal one, which is what is required to underwrite the truth of a proposition about the system (on the orthodox account).[3] The cat paradox is designed to highlight the uncomfortable consequences of this view—effectively by amplifying those consequences so that they affect propositions about macroscopic objects (i.e. cats).

Before delving into the details of a quantum mechanical description of the paradox, let me say briefly how I think we have gone wrong here. According to the orthodox account, the instantaneous quantum state provides a complete description of the system at a time. As the description is complete, the predictions supplied through the Born Rule must match the results of measurements made at that time. The instantaneous state is required, therefore, to underwrite the truth of any proposition about the system—it is assumed to be the 'truthmaker' for propositions P1 and P2. However, the subject of propositions P1 and P2 is not what is happening at an instant (present tense) but what *has* happened (past tense). The paradox arises because the instantaneous state cannot underwrite truths about the past, but the orthodox interpretation maintains that the state provides the only way that propositions about the system can be said to be true.

1.1. Quantum Mechanical Derivation

In terms of quantum mechanics, the propositions P1 and P2 are represented by projection operators (**Prop**). We will write P_{dec} for the projection corresponding to P1 and P_{un} for the projection corresponding to P2. By **Measure** (a), these are mutually exclusive and exhaustive possibilities, and so the proposition P1&P2 is false while the proposition P1$\vee$P2 is true. These projections, then, correspond to mutually orthogonal subspaces that jointly comprise the system's state space, a vector space in which (pure) states correspond to unit vectors.[4] This is easy to say more precisely with the aid of a couple of notational devices.

In Dirac's bra-ket notation, a vector state is a 'bra,' $|\psi\rangle$, and a projection is written as an 'outer product' of the states onto which it projects. For example, the bra $|\psi\rangle$ followed by the 'ket' $\langle\psi|$ forms a projection $|\psi\rangle\langle\psi|$ corresponding to the state $|\psi\rangle$.[5] The projections we are concerned with here can be represented in the same way. We will write $P_{dec} = |decayed\rangle\langle decayed|$ for the projection corresponding to P1, and $P_{un} = |undecayed\rangle\langle undecayed|$ for the projection corresponding to P2. We

[3] Note that we made no use of **Measure** (b) in this derivation. This assumption is problematic because it leads not just to paradox but to inconsistency, i.e. the Measurement Problem. This is discussed in the next subsection.

[4] The state space of a quantum mechanical system is a Hilbert space, $\mathcal{H}$, which is a vector space with an inner product that is complete with respect to the norm induced by the inner product. The subspaces of a Hilbert space are in one-to-one correspondence with the projection operators. A projection operator P is *idempotent*, which is to say that $P^2 = PP = P$. This is why projections may correspond to repeatable measurements.

[5] This is, therefore, a one-dimensional projection onto the 'ray' or one-dimensional subspace spanned by the vector $|\psi\rangle$.

assume they are orthogonal, in which case $P_{un}P_{dec} = P_{dec}P_{un} = 0$.[6] Since these projections jointly include the entire space we have $P_{un} + P_{dec} = \mathbb{I}$, i.e., they sum to the identity. Thus any vector state $|\psi\rangle$ can be written as a *superposition* of orthogonal vector states $|undecayed\rangle$ and $|decayed\rangle$. That is,

$$\begin{aligned} |\psi\rangle &= (P_{un} + P_{dec})\,|\psi\rangle \\ &= |undecayed\rangle\langle undecayed|\psi\rangle + |decayed\rangle\langle decayed|\psi\rangle \\ &= \alpha|undecayed\rangle + \beta|decayed\rangle, \end{aligned}$$

where α and β are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$. These numbers are provided by the inner product $\langle\cdot|\cdot\rangle$.

The instantaneous state, $|\psi_t\rangle$, is supposed to change with time in accordance with an exponential decay law. At the beginning of the experiment, $t = 0$, the atom is undecayed and so the instantaneous state is $|\psi_0\rangle = |undecayed\rangle$, i.e. $\alpha_0 = 0$. After an hour, $t = 60$, there is equal probability that the atom has or hasn't decayed and so we set $\alpha_{60} = \beta_{60} = \frac{1}{\sqrt{2}}$, that is,

$$|\psi_{60}\rangle = \frac{1}{\sqrt{2}}|undecayed\rangle + \frac{1}{\sqrt{2}}|decayed\rangle.$$

This state ensures that probabilities of finding the cat either alive or dead on measurement at $t = 60$ are each equal to a half.

In particular, the Born Rule states that the positive real numbers $|\alpha|^2$ and $|\beta|^2$ correspond to the probability of finding the atom to be undecayed or decayed, respectively, on measurement. For example,

$$\begin{aligned} \Pr(\text{undecayed at } t) = |\alpha_t|^2 &= |\langle undecayed|\psi_t\rangle|^2 \\ &= \langle\psi_t|undecayed\rangle\langle undecayed|\psi_t\rangle \\ &= \langle\psi_t|P_{un}\psi_t\rangle. \end{aligned}$$

This justifies the interpretation of P_{un} as a property (the property of being undecayed) and of $|\alpha_t|^2$ as the probability of finding that property to hold on measurement at t.

In accordance with common sense, **Measure** (b) tells us that, on measurement, the atom is indeed found to be either decayed or undecayed, in which case either P1 is true or P2 is true (making use of **Truth**), while P1&P2 is false and P1$\vee$P2 is true. However, at times $0 < t < 60$ neither the probability of being found to be decayed nor being found to be undecayed is equal to one, and so **Truth** tells us that neither P1 nor P2 is true at times before $t = 60$.[7] This is what leads to the paradoxical conclusion concerning the cat.

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[6] An orthogonal projection is required to be self-adjoint, $P^\dagger = P$.

[7] It remains the case that P1&P2, corresponding to the zero vector, is false and P1$\vee$P2, corresponding to the entire space, is true. However, naturally enough (P1 is true)$\vee$(P2 is true) is false. It should also be noted that neither P1 nor P2 is false before $t = 60$.

As Schrödinger says, the idea of an atomic system existing in this indeterminate state is one that, perhaps, we could get used to. However, there is nothing to stop us from treating the entire setup within the box—cat, detector, hammer, poison and all—as a quantum system. We will write $|dead\rangle$ for the state of the arrangement where the cat is dead and $|alive\rangle$ for the state where the cat still lives. Since the way the situation is set up ensures the cat is dead if the atom has decayed and alive otherwise, the probability of finding the cat dead while the atom is undecayed is zero. According to quantum mechanics, we have following correlated state at $0 < t < 60$:

$$|\psi_t\rangle|cat\rangle = \alpha_t|undecayed\rangle|alive\rangle + \beta_t|decayed\rangle|dead\rangle. \quad (1)$$

This situation has the same interpretation as before: since neither $|\alpha_t|^2$ nor $|\beta_t|^2$ is one, it is neither true that the cat is alive nor true that the cat is dead (before the box is opened).

However, as **Measure** maintains, on opening the box one finds that only one of the incompatible states of affairs has obtained: at $t = 60$ it is true that either the cat still lives, or it is true that it is deceased. The way that **Measure** interacts with **Truth** and **State**, through **Born**, has been taken to be highly problematic. At first it seems as if the reverse direction of **Truth**—if a measurement at time t is certain to reveal the corresponding outcome then the corresponding proposition about the system is true at t—cannot be expected to deliver many truths. Quantum mechanics is, after all, a *probabilistic* theory and so few outcomes can be predicted with certainty. However, since measurements are determinate and repeatable (**Measure** (b)), once a measurement has been made then, according to the reverse direction of **Truth**, the proposition correctly describing the outcome of the measurement is true.

That bears repeating: by making a measurement at some time t, propositions that until that moment lacked a truth value somehow come to acquire a truth value. Since it is hard to deny that the moment the box is opened is a matter of free choice on the part of the experimenter, it seems that quantum mechanics (on the orthodox account) attributes to human observers the power to bring truths into being at a moment of our choosing.[8]

So now we may give the paradoxical nature of the cat's situation its full expression: it is neither true that the cat is dead nor true the cat is alive *until* such time as we deign to open the box and thereby provide to the propositions in question the truth values that they were previously lacking. Thankfully, however, if the cat is found to be dead at time $t = 60$ then **Measure** (b) maintains that it is true at later times that it will remain dead; there is no undead, zombie cat to be found once the box is opened.

The notorious Measurement Problem now arises by considering what the Born Rule would say about the probability of further measurements. Reasoning by means of the forward direction of **Truth**, in conjunction with **Measure**, we decided that, after measurement, the cat must be determinately either dead or alive. So, in case (e.g.) a dead cat was observed we should now attribute a probability of one to finding the cat dead in a future measurement. In order to maintain that **Born**

[8]It does not, however, imply that we can choose *which* state of affairs to bring about, just that there *is* some determinate state of affairs (with probabilities given by the Born Rule).

gives the correct probabilities according to the **State**, the state must now be as follows:

$$|\psi_{t>60}\rangle|cat\rangle = |decayed\rangle|dead\rangle,$$

which is to say that if $t > 60$ then $\alpha_t = 0$. This discontinuous change of state upon measurement is known as the 'collapse of the wavefunction.' If such a change of state did not occur then the premises would not merely be paradoxical but contradictory. In particular, we would be forced to conclude that the state was incorrect and thus find ourselves in contradiction with the completeness of the state (i.e., **State**). To avoid outright inconsistency, von Neumann's infamous Projection Postulate (his Process 1) licenses the observer to change the state in this manner following a measurement (Von Neumann, 1955).

Nonetheless, quantum mechanics is on the usual understanding a *dynamical* theory. Indeed, the Schrödinger equation is supposed to give the dynamics of the instantaneous system state, and left to its own devices it does so in a deterministic manner. This deterministic dynamical evolution apparently leaves no room for the change of state required by the Projection Postulate, i.e. collapse. But without the discontinuous, stochastic change of state provided by the collapse process the theory is apparently in contradiction with experiment: empirically, the outcome of a given measurement *does* depend on the measurements previously performed on the system and their results.

This unhappy state of affairs is known as the Measurement Problem, and without resolving the Measurement Problem it is hard to say just what exactly quantum mechanics is trying to tell us about the nature of the physical world. Because resolving the Measurement Problem means denying (at least) one of the premises above, a solution of the Measurement Problem generally leads to a resolution of Schrödinger's cat paradox (although it may not). But the problems are nonetheless distinct and here I begin with a resolution of the cat paradox which I will then relate to the Measurement Problem, rather than the other way around (as appears to be customary).

2. Avoiding the Paradox: It's About Time

In presenting the paradox I remained faithful to Schrödinger's original presentation, in which the probability of finding the cat alive or dead is given at an instant. I now show how a resolution to the paradox may to be found by explicating more explicitly the non-instantaneous temporal dependance of the health of the cat on the state of the atom. As noted above, in setting up the common sense expectation regarding the cat, Schrödinger made use of a *mixed* tense proposition to say that the health of the cat (present tense) depends on whether or not the atom has yet decayed (past tense). That is, we expect that the present health of the cat depends on the past—on whether or not the cat was exposed to the poison.

This is reflected in the way that, when writing out the instantaneous state of the system, the states attributed the atom have a dependence on the past (either 'undecayed' or 'decayed') while the states of the cat are given in the present tense. However, this was supposed to be an *instan-*

taneous state. I will argue that the failure to faithfully represent this temporal dependence arises from the artificially limited description afforded by the instantaneous quantum mechanical state, and this really lies at the heart of the paradox. To resolve the paradox, I suggest that we reject **Truth** and instead allow the truth (at time t) of propositions about the system to depend on whether or not decay has happened as of time t.

Before addressing quantum mechanics directly, then, let us examine how the paradox could be avoided if Schrödinger's common sense description were correct. Since the propositions in question are tensed, we will need to avail ourselves of some elementary tense logic.[9] Let φ be the proposition that the atom decays (present tense). Then $\mathsf{P}\varphi$ is the proposition that the atom *has* decayed (past tense), where 'P' is the past tense operator. According to the usual interpretation, $\mathsf{P}\varphi$ is true at a time t if and only if φ is true at some time $t' < t$, otherwise it is false.

The proposition that holds of the situation in virtue of the cat's perilous dependence on the state of the Geiger counter and the attached poison releasing mechanism is this: if the atom has decayed (past tense) then the cat is dead (present tense).[10] Let ξ be the proposition that the cat is dead. We may now express Schrödinger's common sense claims as follows:

T1 If $\mathsf{P}\varphi$ then ξ.

T2 If $\neg\mathsf{P}\varphi$ then $\neg\xi$.

It goes without saying that if the cat is not dead then it is alive—it was in good health when put into the box.

Let us assume that at some time t_d the atom decays, and that it does not decay before t_d. Then at all times $t < t_d$ it is true that $\neg\mathsf{P}\varphi$, and at times t' such that $t_c < t'$ it is true that $\mathsf{P}\varphi$. Therefore, T2 ensures that at all times before the atom decays it is true that the cat is alive and T1 ensures that it is true that the cat is dead after the atom decays. What the experimenter finds when she opens the box at some later time t_o depends on whether $t_o < t_d$ or $t_d < t_o$. In either case, she finds out the truth value of the proposition ξ at the time t_o; she does not bring it about (somehow) that ξ *has* a truth value at t_o. In particular, it follows from T1 and T2 quite trivially that at all times it is either true that the cat is dead (ξ) or it is true that the cat is alive ($\neg\xi$).[11]

This analysis captures quite well our common sense intuitions about the situation Schrödinger describes and straightforwardly avoids paradox. In this account of the experiment, the proposition ξ regarding the cat's state of health has a truth value independently of the time that the experimenter chooses to open the box. If the atom were a deterministic classical system then one could use the state of the system at a time $t < t_d$ to predict the time of decay. Schrödinger's use of this example, however, depends on the fact that this is not a deterministic system and so the precise

[9] A model of minimal tense logic is a triple $\langle T, <, p\rangle$, where T is a set of times, $<$ is the 'earlier-than' relation, and p is some time chosen as the present. In what follows I assume that the relation $<$ is asymmetric, transitive and comparable on the set of times, which is the case if they have the structure of the real line.

[10] I take it that the assumption that the cat dies instantaneously is an unproblematic idealization; it is easily removed if a more detailed description is desired.

[11] Note that if $t_o = t_d$ then, according to T1, the cat is dead.

time of decay is left undetermined by the system state. In such a situation, the best we can do is assign a probability to decay before a particular time, in this case t_o. In the presentation above, this probability was provided by the instantaneous quantum state according to the Born Rule.

However, the failure of the system state to predict the time t_d with certainty need not lead to an indeterminacy of the proposition ξ at times earlier than t_d. That is, a failure of determinate prediction need not lead to a failure of determinateness. First, as Earman (1986, pp. 7–13) convincingly argues, determinism is a metaphysical thesis that succeeds or fails independently of epistemic considerations of predictability. Earman's (now standard) definition of determinism is given in terms of agreement, or lack thereof, between physically possible worlds. A physically possible world is a possible world that satisfies the laws of the actual world, and each such world corresponds to an entire history. Earman calls the thesis that any two physically possible worlds which match at a time match at all times *Laplacian determinism*.

If Laplacian determinism fails then there exist two physically possible worlds whose histories agree at some time but disagree at some other time. In that case, the predictions made by an observer in one of those possible worlds with the aid of the laws of the world (common to them both) may fail to determine which world she is in, i.e. which complete possible history of the world describes the actual world. This is not to say, however, that *both* worlds are actual, or neither—the failure of the laws to determine which world she is in is an *epistemic* failure. Following Lewis (1986a, pp. 206–209) I maintain that, metaphysically speaking, the future of a possible world is as determinate as its past. Indeterminism is, therefore, a property of the laws, which (if indeterminism is true) allow two physically possible worlds that agree at one time but disagree at other times.[12]

Moreover, a failure of future-facing determinism need not prevent an individual from learning which world is actual in the fullness of time. In the case we have been discussing (where the time of decay t_d is undetermined by the probabilistic laws) there is nothing to prevent the experimenter from learning which world she is in by leaving the box open and carefully watching the Geiger counter to see when it clicks. In sum, the fact that she cannot predict at precisely *which* time the atom decays does not entail that the atom fails to decay at a determinate time. That is, the experimenter knows that $\mathsf{F}\varphi$ is true without knowing *when* φ is true (where 'F' is the future tense operator).[13] So, although the predictions she is able to make fail to pick out a time uniquely, the *entire* history of the world she is in (i.e., of the actual world) determines whether or not a prediction regarding the time of decay is correct, independently of the way that probabilities are assigned according to the indeterministic laws.

[12]It should be noted that there are alternative accounts of indeterminism that allow for branching to occur within a single world, e.g., (Belnap et al., 2001). According to an account which allows for branching time in this manner, as of the present the future course of events can be regarded as truly indeterminate. As Earman (2008) discusses, his account of indeterminism follows Lewis in allowing only for *ensemble* branching (what Lewis calls *divergence*): instead of one world branching into many worlds we have many worlds that agree at a time coming to disagree at later times. I assume throughout that ensemble branching provides the correct account of indeterminism; see Earman (2006) for arguments against alternative accounts of branching in the context of physics.

[13]Prior (1957, p. 93) discusses a related issue and agrees that "it may be quite definite that it will be the case that *p*, even though there is no time of which it can be definitely said that it will then be the case that *p*." He recommends the use of the future perfect tense as providing an appropriate means to state such truths, i.e. by saying that at some future time it *will have been* the case that *p* was the case.

What I have described here corresponds to a particular kind of indeterminism—a specific way that Laplacian determinism can be violated by the *time* of an event. More precisely, the account of decay times I've sketched describes a violation of Laplacian determinism in the following sense:

Temporal Indeterminism There exists a set of physically possible worlds (i.e. sharing the same laws) each of which corresponds to a distinct, determinate time of decay, $0 < t_d < \infty$, such that any pair of these physically possible worlds match up to time $t = 0$ but diverge afterwards with respect to the time of decay, t_d.

Now, to this it may be objected that the claim that these worlds *match* up to time $t = 0$ risks begging the question: shouldn't it be allowed that differences in the past may determine the time of decay in each world? In Section 5 I argue that quantum mechanics, suitably interpreted, provides good reason to suppose that there can be no such determination of the time of decay by the past. For now, let us accept that the world *could* be indeterministic in this particular sense, which concerns not which event occurs but *when* some event occurs.

This gives the probabilities that describe the time of decay the character of Lewisian chances. As Lewis explains in 'A Subjectivist's Guide to Objective Chance:'

> We have decided this much about chance, at least: it is a function of three arguments. To a proposition, a time, and a world it assigns a real number. Fixing the proposition A, the time t, and the number x, we have our proposition X: it is the proposition that holds at all and only those worlds w such that this function assigns to A, t, and w the value x. This is the proposition that the chance, at t, of A's holding is x. (Lewis, 1981, p. 92)

The proposition playing the role of A in our example concerns whether the atom *will have* decayed at time t_o (future perfect tense). Asserted at some time $t < t_o$, it is true if $t_d < t_o$, false otherwise. Let us take a particular example:

D The atom decays within the first hour.

This proposition D (for decayed) is true if $0 \leq t_d < 60$, otherwise it is false. The truth conditions of the proposition D are, therefore, time-independent and depend only on which world is actual (i.e. the actual time of decay t_d). Also note that the proposition concerns a time *interval*, not an instant.

The chance of D's holding as of $t = 0$ (the time the box is closed) is given according to an exponential decay law with a half-life of an hour. If $t_o = 60$ min then the chance of D's holding as of $t = 0$ is $1/2$. This is Lewis' chance proposition X in this case:

X The chance at $t = 0$ of D's holding at worlds w_X is $1/2$.

It holds at all the physically possible worlds w_X defined by the characteristic that the atom decays at some time in the future, $0 < t_d < \infty$. In each such world, D possesses a determinate truth value because t_d has a determinate value. We can, in fact, define a function $t_d(w_X)$ which takes a value between zero and infinity at each world to which the chance proposition applies.[14]

If such a function exists then at each world w_X there is a determinate time of decay and, furthermore, if the propositions T1 and T2 hold at that world then it follows that it is either true that the cat is alive or true that the cat is dead at all times. These propositions were given a proper expression in terms of the past operator of tense logic. That is, given any time t and a world w_X in which the atom decays at some time t_d, either $\mathsf{P}\varphi$ is true (in which case $t_d < t$) or it is false. If T1 and T2 are true then at any time t either ξ is true or it is false. That is, at all times either it is true that the cat is dead or it is true that the cat is alive. There is, therefore, no need to entertain the notion that probabilistic laws must lead to the indeterminacy of propositions such as P1 and P2.

2.1. Summary and Prospects

In this section I have argued that the probabilistic nature of the decay law need not bear on the determinacy or otherwise of truth-values of propositions concerning the decay time, even those which (at the time they are stated) must be phrased in the future tense.[15] According to my analysis, Schrödinger's cat paradox arises because, on the orthodox account of quantum mechanics, there is no way to assign probabilities to propositions concerning the time of decay in such a way that the time of decay itself can be regarded as the subject of the proposition.[16] However, I maintain that quantum mechanics (properly interpreted) provides no reason to discard the common sense view presented here.

Since I have not yet brought quantum mechanics into the picture, the discussion of this section effectively amounts to a consistency proof of the assumptions that:

1. The laws governing atomic decay are probabilistic (and indeterministic).
2. The cat is either alive or dead at all times (and not neither, nor both).

The idea the time of decay is nonetheless *determinate* (at a world) was crucial to this accommodation. Furthermore, by taking account of tense, it was made clear that propositions (like D, above) concerning the time of decay have to do with decay within a time *interval*, not at an instant.

Despite the fact that propositions about the time of decay are, therefore, eternally true or false (at a world), I argued that probabilities regarding the time of decay still make sense as Lewisian chance propositions (like X, above), indexed to a time and holding of a set of possible worlds. In order to resolve the paradox, then, it will be necessary to show how quantum mechanics leads to probabilistic predictions about the time of decay in such a way that:

[14]The idea here is that we are now considering a collection of models of tense logic in each of which the time of decay, i.e. the time at which φ is true, takes some value $t_d(w_X)$, with $0 < t_d(w_X) < \infty$.

[15]Here I intend to endorse Quine's (2013, §36) attitude regarding future tense propositions without following him in seeking a regimentation of natural language that eliminates tense altogether. As I would have it, the eternalist, four-dimensional point of view provides the truth conditions for tensed propositions, the truth-values of which are, therefore, determined tenselessly.

[16]See Section 6 for further details of how other competing interpretations of quantum mechanics also prevent such assignments of probabilities.

(a) the time of decay is determinate (at a world),

(b) the truth of propositions concerning the time of decay are settled by the (entire) history of the actual world, and

(c) the probabilities concern decay within a time interval, not at an instant.

We are fortunate that in the case of atomic decay we have an alternative description which meets these requirements, provided by the phenomenologically derived exponential decay law. In the next section, I show how the probabilities provided by the exponential decay law, given independently of quantum mechanics, satisfy the requirements (a)–(c) and allow for a resolution of the paradox along the lines suggested here. Furthermore, since they apply on the assumption that the atom *will* decay, I maintain that the chance propositions the law supplies are to be interpreted as *conditional* probabilities. This leads, in Section 4, to a proposal to derive these probabilities from the quantum state in precisely the same way.

3. Exponential Decay: Phenomenological

The resolution of the paradox I advocate requires that we reject the implicit assumption of the orthodox account that the probabilities supplied by the quantum mechanical state apply to the results of instantaneous measurements. An examination of how the phenomenological exponential decay law assigns probabilities to intervals of time (and thus chances to propositions like D) will provide some useful insights that I will later apply to quantum mechanics. Let us now consider how the decay law arises from phenomenological (i.e., empirical) considerations, independently of quantum theory.[17]

Taking a uniform sample of some radioactive isotope—radium, say—one observes that the number of nuclei that will decay in a given interval of time is proportional to the original number of nuclei in the sample. Furthermore, the proportion of the original sample of N_0 nuclei that remains after a time t is observed to follow the simple rule

$$\frac{N_t}{N_0} = e^{-t/T} = e^{-\lambda t},$$

where T is the mean lifetime of the parent nuclei.

The half-life, $\tau_{1/2}$, is the time it takes for half the sample to decay, i.e., the value of t such that $e^{-\lambda t}$ is equal to a half. The half-life of radium, for example, is approximately 1600 years. The expected proportion of the sample that remains at time t, then, can be equivalently expressed as

$$\frac{N_t}{N_0} = e^{-(\tau_{1/2} t)/\ln 2} = e^{-\lambda t}.$$

[17] In case of confusion, note that I am using 'phenomenological' in the physicist's sense, not the philosopher's. That is, phenomenological operates here as a synonym of experimental, or empirical.

But if our sample consists of a *single* radium atom then this fraction must be interpreted as a *probability* of decay after time t. And taking this point of view as fundamental explains the observed behavior of a larger sample: each atom in the sample has the same probability of decay, given by the same exponential law.[18]

To calculate the probability that the time of decay of a single atom, t_d, lies within some time interval $[a, b]$, where $a \geq 0$ and $b > a$, we integrate the corresponding *probability density*, $\lambda e^{-\lambda t}$, as follows:

$$\Pr(a \leq t_d < b) = \int_a^b \lambda e^{-\lambda t} dt. \tag{2}$$

If, as in the case of Schrödinger's cat, the half-life is an hour, then this returns the expected result that the probability of decay in the first hour is a half, $\Pr(0 \leq t_d < 60) = 1/2$, and the probability of decay in the second hour is a quarter, $\Pr(60 \leq t_d < 120) = 1/4$, and the probability of decay in the third hour is an eighth, $\Pr(60 \leq t_d < 120) = 1/8$, and so on. It can easily be seen that this assignment of probabilities is properly *normalized*, in that $\Pr(0 \leq t_d < \infty) = 1$, i.e. such that the atom is certain to decay at *some* time in the future.

This shows that the phenomenological law assumes that the following future tense proposition is true at $t = 0$: the atom will decay (i.e., $\mathsf{F}\varphi$). One could even say that it *entails* that the atom will decay. The way I'd prefer to think of it is that this is an assignment of conditional probabilities: the chances provided by the decay law apply *given that the atom will decay*. That is, they give probability one to the event of decay at *some* future time, and apply just in case the corresponding proposition is true.[19] The chance propositions provided by the decay law, then, concern *conditional* probabilities: in assigning these chances at time t we condition on the event that decay occurs during the interval $[t, \infty)$.

3.1. Chance and Times

Although Lewis does not discuss these cases, one might worry that this inclusion of information about the future in a proposition about the chances at t amounts to a use of inadmissible evidence. This phenomenon of assigning chances through conditionalization on future events is far from unusual, however. For example, the assignment of equal probabilities to the results of a coin toss describes the chances only in worlds where the coin is thrown fairly, and, indeed, only in worlds where it is in fact thrown at all. This also amounts to a condition on a future event: the event that a fair coin toss takes place as described.[20]

Take the coin toss I am about to do as I write. I have in my wallet a quarter, which I intend to take in my hand, position over my thumb, and give a good flick, letting it fall on the floor by my

[18]The foregoing follows van Fraassen (1991) rather closely, who notes that without this assumption we would be led to strange conclusions about odd numbered samples of atoms.

[19]Note, however, that the empirical success of the exponential decay law gives us very good reason to regard this proposition as being true (so long as the sample is as described).

[20]As Hoefer (2007, pp. 553–555) argues, there is no need to define the admissible evidence with reference to time: what really matters to Lewis' (1981) Principal Principle is that the definition of admissible evidence ensures that only sort of evidence that affects our credence about outcomes comes from credence about chances.

side. To the event of the coin's landing heads I assign a probability of a half, and to its landing tails I assign a half, likewise. What events, precisely, am I talking about here?

Some characteristics stand out: they are future events, and they are mutually exclusive. If the coin toss I am about to perform lands heads, it will not land tails. But what if 'the coin toss' simply doesn't occur? I could decide right now, in fact, not to toss the coin at all. How could I have assigned meaningful probabilities to the outcomes of an coin toss that doesn't happen? Or, what if the quarter is not to be found in my wallet and I use a dime instead? Is this the same coin toss that I assigned these probabilities to?

These worries are not deep, and they are easily remedied. In order to be well-defined, chance propositions concerning future events have an implicit conditionalization in place—they are really *conditional* probabilities. Given that a fair coin toss will occur, I assign to each of the mutually exclusive alternative outcomes probability half. If no such event occurs, then there is nothing to which my probability assignment refers.[21] When I assign probabilities to future events, I do so only on the assumption that there will in fact occur an instance of the sort of thing that falls under the description of the arrangement to which I take my probabilities to apply.[22] Hoefer (2007) calls these probability-apt situations *chance setups*.

In other words, it is not implausible that *all* probability assignments—all true chance propositions—concern conditional probabilities. Hoefer regards objective chances as *properties* of a chance setup—a situation such as a fair coin toss or roulette wheel spin—and regards this as a tenseless notion. As he puts it:

> Typically chance setups involve a temporal asymmetry, the 'outcome' occurring after the 'setup' conditions are instantiated. But in no case do the categories of past, present or future (as opposed to before/after) need to be specified. (Hoefer, 2007, p. 555)

However, whereas Hoefer prefers to regard chances as time-independent properties of chance setups, I maintain that Lewis' assignment of chances to times captures something important about the practice of assigning probabilities to the time of an event. In such cases the admissible evidence does indeed change with time, and we are right to say that the chance that a given event occurs during some (tenselessly given) time interval changes as time progresses. The point of view I prefer combines both insights: I maintain both that chances are conditioned on the existence of a chance setup to which they apply *and* that chances hold at a time. Preserving both these insights allows us to accommodate one of the more counterintuitive aspects of the radioactive decay process.

Considering again the exponential decay law, we see that the probabilities it supplies exhibit a striking temporal independence of the decay process: the probability that a particular nucleus will undergo radioactive decay during a given time interval does not depend on the age of the nucleus

[21] And, as Hoefer (2007) points out, there is no reason I should not bet on the outcome of a fair coin toss that has already taken place (and of whose outcome I am ignorant). When it comes to assigning probabilities to outcomes, he argues, the coin toss in the past is no less worthy of chance than the coin toss in my future—especially if we regard the future as settled (i.e., determinate); in that case there can be no difference at all.

[22] I did perform the coin toss and it landed tails, in case you were wondering.

or its history. In particular, given that an atom has not decayed as of time t, the probability that it will decay in the *next* hour (or minute, or second) is always the same, no matter what value t takes.

To see how this works, imagine that we opened the box after an hour, found the cat alive and concluded that the atom did not decay. In that case, we would update our probabilities to reflect the fact that the probability of decay in the first hour is now zero, $\Pr(0 \leq t_d < 60) = 0$. Having done so, what is the probability of decay within the second hour, $\Pr(60 \leq t_d < 120)$? Note that we cannot leave the previous assignment of values as it is since then we would not have a set of probabilities at all. That is, to qualify as probabilities they must sum to unity, but as it stands we have the following:

$$\Pr(0 \leq t_d < 60) + \Pr(60 \leq t_d < \infty) = 1/2.$$

In practice, we resolve this difficulty by assigning to the probability of decay within the second hour a half, $\Pr(60 < t_d < 120) = 1/2$, and to the probability of decay within the third hour a quarter, $\Pr(120 < t_d < 180) = 1/4$, and so on, whereas before this 'renormalization' these were a quarter and an eighth, respectively. The renormalization process amounts to using the rule (2) to calculate the new probabilities under a reassignment of labels to times. That is we, relabel the times so that what was $t = 60$ becomes $t' = 0$, i.e. $t' = t - 60$. This amounts to a shift of the probability assignments forward by an hour.

This leads to the conclusion that if at any time t the atom has not yet decayed then the probability of decay within the *next* hour (i.e., the interval $[t, t + 60]$) is a half. This temporally dependent, amorphous character of the values of the probabilities for decay within a given interval would be inexplicable if the probabilities were somehow meant to attach to properties possessed by the system at particular times—if they were properties of the chance setup alone. However, their dependence on subjectively possessed information (or the admissible evidence, as Lewis would have it) makes perfect sense if these probabilities are conceived of as Lewisian chances.

Before the experiment begins, at time $t = 0$, the chance of decay during the second hour is a quarter. Call this chance proposition X_0. It holds in all worlds in which the atom decays at some later time, $0 < t_d(w_X) < \infty$. But in the worlds under consideration here (where the cat is found to be alive at $t = 60$) the proposition that the atom decays during the first hour is false. After an hour, the experimenter learns this fact and updates the chances accordingly since this is now admissible evidence.

Call the following proposition X_1: 'the chance at $t = 60$ of decay during the second hour is a half'. The chance proposition X_1 holds at all worlds w_{X_1}, i.e. those worlds where $60 < t_d(w_X) < \infty$. There is no contradiction involved in saying that in *these* worlds that the chance of decay during the second hour is a quarter at $t = 0$ but a half at $t = 60$. So X_1 does not hold in those worlds in which the atom did decay in the first hour, i.e. where $0 < t_d(w_X) < 60$; in *those* worlds the chance at $t = 60$ of decay during the second hour is zero.

That is to say, the chance proposition X_0 holds at a distinct set of worlds from the chance proposition X_1 (and at a distinct time). Lewis explains this by saying that the admissible evidence changes with time, and with it, here, the chances concerning decay within a given time interval, which leads to variation in the worlds to which the chance proposition applies. There is one chance proposition that doesn't change with time, however. The chance that the atom decays at some time $0 < t_d < \infty$ is always one, no matter what the time, and the corresponding proposition is true at every world to which the chances apply.

The key to resolving the paradox then, I claim, is to assign quantum mechanical probabilities to time intervals (the times at which the atom might decay) in a way that respects the normalization of the phenomenological decay law. That is, in such a way that the proposition that the atom *will* decay can be said to be true at $t = 0$. In other words, the desired probabilities are *conditional* probabilities, conditioned on the fact that the atom will decay at some time $t_d(w_X) < \infty$. This is not obviously the case with the probabilities supplied by the Born Rule according to the orthodox account of quantum mechanics.

4. Exponential Decay: Quantum Mechanical

Having shown how the phenomenological law allows for the assignment of probabilities to time intervals, let us return to quantum mechanics. I now demonstrate that the quantum state can be used to supply probabilities for the time of decay in the same way as the phenomenological decay law. I approach this problem using the exponential decay law as a guide, coupled with the analysis of chance propositions developed above. The key interpretative assumption I make at the outset is that a time-indexed (Heisenberg picture) projection corresponds to a present tense proposition about an event occurring at that time (here, the decay of the atom).

Essential to this idea is the observation that there is no need to consider only instantaneous states in quantum mechanics. The availability of an equivalent non-instantaneous, tenseless description of the state of a quantum mechanical system is guaranteed by the equivalence of the Heisenberg and Schrödinger pictures of time evolution. So far we have only discussed the Schrödinger picture, in which the state of the system is given instantaneously and varies with time according to the Schrödinger equation. In the Heisenberg picture, however, the state of the system is given independently of time and it does not dynamically evolve.

The two pictures are equivalent because the *observables* in the Heisenberg picture (which are in one-to-one correspondence with the Schrödinger picture observables) evolve with time (according to the Heisenberg equations of motion). This means that each picture supplies the same predictions for the results of an instantaneous measurement of an observable.[23] The use of the Schrödinger equation in quantum mechanics is, therefore, inessential, and historically the use of the Heisenberg picture (in the matrix mechanics of Heisenberg, Born and Jordan) antedates the use of the Schrödinger equation. My accommodation of tensed propositions like D—which refer

[23] In particular, if $U_t = e^{-itH}$ is the unitary group uniquely generated by H, the Hamiltonian (a self-adjoint operator), then the Schrödinger picture observable A (a self-adjoint operator) corresponds to a family of Heisenberg picture observables, $A(t) = U_{-t}AU_t$. The pictures are equivalent because the expectation of a Schrödinger picture observable at time t is $\langle A \rangle_t = \langle \psi_t | A\psi_t \rangle = \langle \psi | U_{-t}AU_t\psi \rangle = \langle A(t) \rangle$, i.e. they return the same expectation value at time t for any observable.

to time intervals rather than instants—to quantum mechanics is best thought of in terms of the time-independent Heisenberg picture state.

Let $\varphi(t_d)$ be the proposition that the atom decays at time t_d. It corresponds, I claim, to a Heisenberg picture projection $P_\varphi(t_d)$. If the Schrödinger picture state of the system is $|\psi_t\rangle$ at time t then let the time-independent Heisenberg picture state be $|\psi\rangle = |\psi_0\rangle$. A chance proposition like X concerns the probability that decay occurs at time $0 \leq t_d < t_o$. In analogy with the phenomenological decay law, I propose that this probability is to be given by an integral over an instantaneous probability density.

This probability density is given in terms of the function $f_\varphi(t) := \langle\psi|P_\varphi(t)\psi\rangle$, which is a real-valued function of t determined by the Heisenberg state $|\psi\rangle$ and the time-dependent projections $P_\varphi(t)$. On the orthodox account, the value of this function is interpreted as the probability of finding the proposition φ to be true if the corresponding projection is (instantaneously) measured at time t. Instead, I propose to interpret this function as a probability density for the time of decay in the following way.

As was the case with the phenomenological decay law, we need to ensure that the probabilities assigned to time intervals are properly normalized so that the event of decay at time $0 \leq t_d < \infty$ is assigned probability one. This event corresponds to the following integral:

$$\langle T\rangle := \int_0^\infty f_\varphi(t)dt = \int_0^\infty \langle\psi|P_\varphi(t)\psi\rangle dt.$$

Assuming the integral converges, $\langle T\rangle$ is a positive real number which we can think of as giving (something like) the mean time of decay. Just as we used $\lambda = 1/T$ to normalize the exponential decay law $e^{-\lambda t}$ and form a probability density $\lambda e^{-\lambda t}$, I propose to use $\mu = 1/\langle T\rangle$ to normalize the function $f_\varphi(t)$ to form a probability density $\mu f_\varphi(t)$.

This gives us the following quantum mechanical expression for the probability that the time of decay lies within a time interval $[a, b]$:

$$\Pr(a \leq t_d < b) := \int_a^b \mu f_\varphi(t)dt = \int_a^b \frac{\langle\psi|P_\varphi(t)\psi\rangle}{\int_0^\infty \langle\psi|P_\varphi(t)\psi\rangle dt} dt. \tag{3}$$

Quantum mechanics, then, would precisely replicate the exponential decay law in this particular case if we found that $f_\varphi(t) = e^{-\lambda t}$. In that case, we would see that $\langle T\rangle = 1/\lambda$ and thus

$$\Pr(a \leq t_d < b) = \int_a^b \lambda e^{-\lambda t} dt,$$

as before.[24] The crucial point to emphasize here, though, is that the expression (3) is valid for just about any quantum state and time-indexed projection $P_\varphi(t)$, representing the occurrence of any event at all.

[24] Note that this is exactly what we find if the function $f_\varphi(t)$ leads to the appropriate Breit-Wigner amplitude (Blank et al., 2008, p. 341), which can be regarded as an approximation of the decay amplitude of a Gamow state (de la Madrid, 2012), introduced by Gamow in 1928 to describe alpha decay.

This analysis forms the basis of my resolution of the paradox.[25] If we regard these probabilities as chances, valid at $t = 0$, then we may preserve the interpretation I set out in the previous section. In the case of Schrödinger's cat, the chance propositions given by this expression apply to just those worlds w_X in which the atom decays at some time $0 \leq t_d(w_X) < \infty$, and if T1 and T2 are true at those worlds then propositions about whether or not the cat is dead have truth values at all times. In general, however, the assignment of chance propositions may change if further conditioned on the non-occurrence of the event at some time $t_o < t_d(w_X)$. There is no contradiction in this because the chance propositions, so conditioned, apply to distinct sets of worlds at different times.

5. Some Interpretative Posits: Stochastic Histories

To follow through the resolution of the cat paradox suggested here it is clear that much of the orthodox account of the theory will have to be discarded, particularly **Truth**. However, I aim to keep as much of it in place as I can, and in particular I change the mathematical formalism as little as possible. On my view it is just the interpretation of the probabilities—as applying to the results of instantaneous measurements—that is at fault. The major interpretative modification I wish to suggest is that we allow time-indexed propositions (corresponding to projections) to be interpreted as possible events occurring at some definite time. This replaces the idea that they correspond to instantaneous measurements or properties. Although a relatively minor change, this allows for the truth-values of other propositions (at a time) to depend on whether or not such an event has yet occurred.

In addition, I claim that we are justified in believing certain propositions about the future to be true in many experimental situations, regardless of what the orthodox account says. In the case considered here, the proposition was that the atom will decay at some time (i.e., $\mathsf{F}\varphi$). The probabilities so assigned are, therefore, conditioned on the truth of this proposition. By allowing the probabilities to apply to the time of decay, and in particular the proposition that the time of decay lies within a certain interval, I was able to apply an interpretation of the probabilities as Lewisian chances, conditioned on the existence of a chance setup (in Hoefer's terms). This allows the tense-based analysis of Section 2 to remain in place, and thus at any time t either it is true that the atom has decayed or it is true it has not. This justifies my contention that it is either true that the cat is dead or true that the cat is alive at all times, and thus resolves the paradox.

To preserve and extend my quantum analysis of radioactive decay we will require some new interpretative principles, replacing those of the orthodox account. In its stead, I claim that quantum mechanics applies to times in the following way:

State* A history defines a unique quantum mechanical state giving the best possible information about a system at a time.[26]

[25] For details of how this way of assigning probabilities can be accommodated by the usual mathematical formalism of quantum mechanics, see the Appendix.

[26] In particular, I assume that this state serves to screen off the probabilistic dependence of future events on past events.

Chance The state provides time-dependent chances for the time of an event which are conditioned on the future occurrence of that event.

Prop* A time-indexed proposition $\varphi(t)$ corresponds to

(a) a time-indexed (Heisenberg picture) projection $P_{\varphi}(t)$

(b) an event occurring at t.

Truth* An outcome obtains at time t at a world w *iff* a proposition $\varphi(t)$ is true at w.[27]

Measure* The possible times of an outcome are

(a) mutually exclusive and exhaustive

(b) determinate (at a world).

To these posits I attach the epithet: the Stochastic Histories interpretation.

Although these interpretative posits are quite distinct from those of the orthodox account that we began with, the mathematical formalism is almost entirely unchanged. The major formal difference is the use of Lüders' Rule, which supplies conditional probabilities, rather than the Born Rule—but Lüders' Rule is just the standard extension of the Born Rule to accommodate conditional probabilities (Bub, 1977). (See the Appendix for details.)

What cannot survive, however, is the orthodox interpretation's use of quantum logic, in which propositions about the system represent the possible properties of a system, given instantaneously. I reject this conception of propositions about the system in favor of an interpretation of projections in terms of events and, in particular, Heisenberg picture projections as corresponding to the *occurrence of an event at some time*.

This is in accord with the recent suggestion of distinguished theoretical physicist Rudolf Haag:

> What do we detect? The presence of a particle? Or the occurrence of a microscopic event? We must decide for the latter. [...] [T]he standard use of the term "observable" does not really correspond to the needs of collision theory in particle physics. We do not measure a "property of a microscopic system", characterized by a spectral projector of a self-adjoint operator. Rather we are interested in the detection of a microscopic event. The first task is to characterize the mutually exclusive alternatives for such an event. (Haag, 2013, p. 1310)

In the case I am considering here, the mutually exclusive alternatives are the *times at which an event can occur*, which I have assigned probabilities to by referring to the time intervals that these times may lie within.

Another distinctive feature of the account of quantum theory advocated here is the way that, according to **Truth***, a proposition about the system can be true independently of what the state

[27] The interpretation of $\varphi(t)$ in terms of my modalized tense logic is as follows: $\varphi(t)$ is true at a world w *iff* φ is true at time t at w.

says (this is what allows us to avoid the Measurement Problem). The difficulties of the orthodox account arose because of the demand that the state should underwrite the results of measurement. This turned out to be incompatible with the deterministic dynamics of the system state. By denying that the quantum state needs to play that role, we avoid this conflict.

As Bub puts it:

> The occurrence of these events is only in conflict with the evolution of the quantum pure state if the quantum pure state is assumed to have an ontological significance analogous to the ontological significance of the classical pure state as the 'truthmaker' for propositions about the occurrence and nonoccurrence of events—in particular, if it is assumed that the quantum pure state partitions all events into events that actually occur, events that do not occur, and events that neither occur nor do not occur, as on the usual interpretation. (Bub, 2011, p. 261)

The idea that the quantum state should play such a role is called 'the second dogma of quantum mechanics' by Bub & Pitowsky (2010), and they reject it.[28] I also reject the assumption that the quantum state is a truthmaker, in this sense. On my account, facts about the world (i.e. the history of the actual world) make propositions true, and by that I mean to say that the truthmaker of propositions about an event is the event itself, which exists concretely, eternally, and independently of our description of it.

A proposition such as $\varphi(t)$ is made true, then, by the concrete existence of events; of the things that happen. The sum total of all events that are (tenselessly) actual is the history of the (actual) world. Thought of as a Humean mosaic, i.e. with no necessary connections between the events, a specification of such a history can also represent a possible world; a way the world could have been. That is, these histories represent other ways the world could have been but isn't, and also ways that it might yet be (for all we know). Some of the histories are compatible with the laws of the actual world and those are the physically possible worlds (in Earman's sense).[29]

I have proposed that, as befits a probabilistic theory, the quantum state provides chances for the time of an experimental outcome, just as the orthodox account allows for the state to provide chances for which outcome obtains on measurement at a time t. To regard those outcomes as constituting a chance setup on the orthodox account, however, requires conditioning on the fact that measurement occurs at some particular time. My proposal allows for us to condition on the fact *that an outcome obtains*, without committing ourselves to a particular time at which it obtains.

[28]The first dogma is J. S. Bell's idea that the word 'measurement' should not appear in the foundations of a fundamental physical theory. Unlike Bub and Pitowsky, I absolutely agree with Bell on this point.

[29]This assertion may lead to worries about Lewis' (1986b) 'big bad bug,' arising from the conjunction of Humean supervenience, the Mill-Ramsey-Lewis account of laws, and Lewis' Principal Principle. Although I cannot address the worry in full here, my contention is that Hoefer's (2007) 'third way on objective chance' provides the means to avoid the bug by conditioning chances locally on the existence of a chance setup, as I have done throughout. Hoefer's key insight is that chances need not be defined globally with reference to the relative frequencies of events at large but can instead be regarded as "constituted by the existence of [local] patterns in the mosaic of events in the world" (Hoefer, 2007, p. 463), which he regards as stable, regularity generating 'stochastic nomological machines.'

Surprisingly, these features are quite difficult to replicate within other interpretations of quantum mechanics.

6. Situating the Proposed Resolution

In essence, the problem highlighted by Schrödinger's cat is that (on the orthodox account) propositions about the system only come to possess truth values on measurement. According to the orthodox account of quantum theory, then, the propositions to which chances are assigned are outcomes of measurements. Therefore, a quantum system only qualifies as a chance setup if a measurement is performed. This is reflected by the way that, on the orthodox account, when I assign a probability to the cat being alive, in fact I am assigning a probability to finding the cat alive *on measurement*, i.e. these probabilities are conditioned on the fact that a measurement is made.

In the case of Schrödinger's cat, where the probabilities change with time, the orthodox account invites us to condition on the fact that a measurement is performed at a particular time (and no measurement was made previously). These probability assignments are, then, (i) valid at a time, and (ii) conditioned on the fact that a measurement occurs at that time. The difficulties with Schrödinger's cat arise precisely because the time of measurement is out of the scope of the description of the system by quantum mechanics; it is something that we must put in by hand.

I propose instead that we retain the common sense account of Schrödinger's cat, which essentially says that, while it is possible at $t = 0$ that the atom will decay at *any* later time, it will, in the fullness of time, in fact decay at some definite time $t_d > 0$. Thus the cat is dead at times before t_d and alive at times after t_d. Here, the chance setup concerns the *time* of the event, and the outcomes to which probabilities apply take the form of 'decay within time interval $I = [0, t)$' (i.e., corresponding to propositions such as D). What is it about the quantum description that leads others to deny these claims?[30]

6.1. Everettian Approaches

To see why such a probability assignment is hard to square with quantum mechanics, the Everett interpretation provides an instructive first case. According to Everett, it will be true that the atom has decayed and true that it has not; likewise it will be true that the cat is alive and true the cat is dead. To avoid contradiction, Everettians maintain that the cat that is alive and the cat that is dead are in fact *two different cats*.

The two terms that appear in the quantum superposition (1) correspond, therefore, to two distinct equally actual states of affairs and the weighted coefficients that attach to them determine the credence I should give to the proposition that I will see a live or dead cat on interaction with the system (i.e., 'measurement'). The 'I' to which this statement refers, though, will have also bifur-

[30]In what follows I limit myself to discussion of only interpretations that do not modify the mathematical formalism of quantum mechanics. My view is that the interpretation of the formalism is at fault, not the formalism itself. See Pashby (2014b) for some critical remarks on GRWf in this context.

cated. At the beginning of the hour, the live cat branch is most 'weighty' and as time progresses it becomes less and less weighty until an hour has passed, at which point both are equally weighty.[31]

The Everettian is quite within her rights to maintain that the notion of measurement has been defanged by this maneuver. No matter what time I interact with the system I will bifurcate precisely according to the weights at that time and (all being well) my credences will match that weighting. The time of interaction has no role to play in bringing it about that the experiment has an outcome because, according to the Everettian, *no experiment ever has* an *outcome*.

But note that nothing in this picture corresponds to the *time* at which the atom decays and the cat dies. In *neither* branch is it true that the atom decayed at a particular time: it either has decayed, or hasn't decayed, and at each time there exist two versions of the experimenter happy to report that (relational) fact, as appropriate to the divergent branches they occupy. So, on the Everett interpretation of quantum mechanics, it is hard to say what it is exactly that I am assigning probabilities *to* when I say that the chance of decay within the next hour is a half. The time of decay has become a meaningless notion, replaced, it seems, by a measure of the tendency of an observer to report (when asked) that decay has occurred.

However, this may be too quick: we may still be able to make sense of a time at which *branching* occurred. A potential problem with this idea is that, according to the Everettian interpretation of the state (1), branching occurred as soon as the box was shut (at which point there was a non-zero weight attached to the 'dead cat' branch). However, in the more recent, pragmatically oriented Everettian approach advocated by Wallace (2012), there is a certain sense in which the Everett interpretation does allow a rough and ready determination of the time at which branching occurs through consideration of the interaction of a system with its environment.

According to Wallace's account, that is, there is a certain pragmatic sense (valid For All Practical Purposes) in which branching may be described as a dynamical process, and accordingly so too may the (approximate) time of branching be specified.[32] The dynamical process in question is known as *decoherence*, and the key idea of the recent Everettian consensus is that the branching structure of the wavefunction is to be given with respect to the dynamics of the system-environment interaction.

In particular, if at some time there exists a set of projections on the (reduced) state space (corresponding to propositions about the system but not the envrionment) such that the state is approximately orthogonal with respect to them (at some time t) then one can regard each of those projections as representing an event that occurs at t in a distinct (and roughly non-interacting) branch. Although the universal dynamics does not pick out *which* event occurs (occurrence being here a branch relative affair), it does provide a notion of *when* those events occurred.

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[31] I do not use the word 'probable' here because the connection of branch weights to probabilities (i.e. chances) requires independent argument—if both outcomes occur then the conventional account of probability does not apply. Contemporary Everettians aim to replace the Born rule with a decision theoretic justification for regarding branch weights as subjective probabilities that obey Lewis' Principal Principle, and thus qualify as objective chances. See (Wallace, 2012, Ch. 4–5) for more details.

[32] One of the problems with this approach, admittedly, is that there is no precise way of counting branches, and thus no precise way of knowing when branching occurs (Wallace, 2012, p. 99–102).

This idea could be thought to supply an answer to the puzzle I raised about the facility of the Everett interpretation to make room for a meaningful representation of time of decay. However, the fact that the time of branching is given by the *deterministic* dynamics acting at the universal level acts to rule out the idea that the time of branching could play the role of the time of decay. At least, the sort of uncertainty than an observer has about which outcome they will experience cannot be the same sort of uncertainty they may have about *when* an outcome obtains.

In particular, the way that times are determined by the dynamics of an Everettian 'multiverse' means that the sort of probabilities that I am concerned with, regarding the *time* of an event, apply to an ensemble of multiverses, not a collection of branches of one multiverse. Instead, the many worlds of Everettian quantum mechanics have (approximately) the structure of a branching time series, with the times at which new branches are added being given by the universal dynamics. There are no Everettian worlds, then, that vary according to the time at which some event occurs. These are not the possible worlds of my resolution of Schrödinger's cat paradox.

6.2. Von Neumman's Projection Postulate

The Everettian maintains that the deterministic evolution of the system state proceeds unrestrictedly at the universal level while the apparently indeterministic character of experimental quantum physics is preserved at the subjective level (at the cost of the multiplicity of the observer). In contrast, the conventional account of measurement, which endorses von Neumann's Projection Postulate, punctuates the deterministic evolution of the system state with measurement-induced collapse. The determinism on offer here is, therefore, on shaky ground since it is prone to being overturned at the whim of the observer.

Moreover, the *in*determinism on offer is difficult to articulate without essential reference to measurement, and thus the observer. In particular, it is not just *which* measurement is being performed that matters, but also the *time* at which measurement occurs. The system used to put Schrödinger's cat in peril was particularly well chosen to highlight this feature because of the way the probabilities change with time. If the time of measurement is assumed to be the free choice of the observer then the conclusion that the observer herself brings about collapse becomes unavoidable.

Von Neumann's (1955) characterization of collapse was as a process that brings about the transition of the system from a pure state (given by a one-dimensional projection such as $|\psi\rangle\langle\psi|$) to a statistical mixture (corresponding to a normalized sum of such projections $W = \sum_n c_n |\psi_n\rangle\langle\psi_n|$, i.e. a density operator). According to von Neumann, the deterministic evolution of the system state is suspended at the moment of collapse, and after that time the probabilities for finding a given outcome may be thought of in epistemic terms. That is, following collapse the statistical mixture is apt to be given an ignorance interpretation: we are at liberty to regard *some* outcome as having obtained and, in that case, propositions about the health of the cat have a determinate (albeit unknown) truth value.

However, because the (epistemically interpreted) probabilities are fixed once collapse has occurred, the time dependence of the coefficients describing the radioactive decay process entails that, to be empirically correct, the time of collapse must equal the time of measurement. According to von Neumann, however, the time of measurement must of necessity be given externally to the theoretical description of the system according to quantum mechanics. Without the means to assign a probability distribution to the time of an outcome within the theory, we have thus reached the conclusion that the observer brings about collapse at a time of her choosing.

6.3. Van Fraassen's Modal Interpretation

Modal interpretations provides an alternative reconciliation of the deterministic dynamics with the indeterministic character of experimental outcomes that cleanly separates (and retains) both aspects. Van Fraassen's pithy statement of the core position is as follows:

> By a modal interpretation I mean one based on the premise that the propositions studied in quantum logic, which can be used to express facts about the quantum mechanical state of a system, are modal: they give information first and foremost about what can and what must happen, and only indirectly about what actually does happen. (van Fraassen, 1981, p. 229)

The propositions of quantum logic are just the projections of (what I have called) the orthodox account. In addition to the usual (instantaneous) dynamical state, however, van Fraassen (1981) proposed that each system has a value state. The possible value states correspond to different ways the system could have been—possible worlds—and to the dynamical state, therefore, corresponds an ensemble of possible worlds, at most one of which is actual. Van Fraassen's modal interpretation replaces the notion of **Truth** by a distinct semantics that makes reference to the value state rather than the dynamical state, in which case the quantum state (considered without the value state) is incomplete.

However, the modal interpretation cannot make room the time of an event to be both determinate and to vary between possible worlds, and as a result fails to accommodate my resolution of Schrödinger's cat paradox. The first difficulty is that van Fraassen's 'Copenhagen Variant' adopts a semantics that sticks rather closely to the idea that a quantity takes a particular value only following a measurement of that quantity. The second difficulty is technical: there is provably no way that a value state can determine the time at which an event occurs (at a world). Let's take the conceptual difficulty first.

As van Fraassen admits, because the value state is only required to underwrite the results of measurements at the times they actually occur,

> in the case of Schrödinger's Cat, the account I gave above [according to the Copenhagen Variant] is logically compatible with the cat being sometimes alive and sometimes dead, between the triggering of the device and the opening of the box. (van Fraassen, 1991, p. 297)

As Ruetsche (2003) puts it (summarizing Maudlin):

> a world of which QM, so interpreted, is true is a world in which possessed values may fluctuate wildly among the values the interpretation deems possible—*even if the quantum states of the systems at issue don't change at all!* (p. 30, original emphasis)

This strikes me as a partial resolution of the paradox at best. Common sense requires not just that the relevant propositions take *some* truth value but also that a dead cat *stays dead*. In my account of the situation, this problem was avoided by means of an explicit dependance of the health of the cat on the time of decay. In giving a modal semantics that makes no room for the notion of the *time* of an event, van Fraassen cannot provide the same reassurance.

On my diagnosis, then, this difficulty results from the fact that the possible worlds of van Fraassen's modal account are not *histories*. Since van Fraassen retains the underlying assumptions—taken unchanged from the orthodox account—that the values taken by observables are (i) determined by measurements at times given independently of the dynamics, and (ii) to be given instantaneously with no reference to other times, he is unable to escape the problematic conclusion that only measurement fixes the truth value of a proposition about the system. My contention is that we can avoid such puzzling and counterintuitive fluctuations in truth values by allowing the truth values of tensed propositions to depend on the history of a world rather than its instantaneous state.[33]

As I demonstrated in Section 2, the fluctuations in truth values could be avoided by making reference to the time of decay, or (equivalently) ensuring that propositions about times have determinate truth values (at a world). However, van Fraassen's value state provides no way to specify whether or not a proposition regarding the time of an event is true at a possible world. This complaint, about the limited capacity of the value state to underwrite facts about the time of decay, also applies to von Neumann's idea that measurement corresponds to the transition of a pure state to a statistical mixture.

The problem is this: the existence of a mixed state describing a particular set of possible experimental outcomes (one of which is actual) relies on the existence of a corresponding set of mutually orthogonal projections. However, where the outcomes are *times*, there is provably no such set of projections that would correspond to propositions (like D) concerning times. But if no such projections exist then the value state is incapable of assigning to a possible world a definite time of decay.

The mathematical result from which this conclusion follows is known as 'Pauli's Theorem.' It has often been taken to show that time has something of a special status in quantum theory. The interpretation I would like to give that special status here is this: the idea that the *time* of an event (like decay) is governed by an indeterministic process is incompatible with the idea that the

[33]The contrast between my proposal and van Frasssen's modal interpretation could be phrased as follows: whereas van Fraassen takes probabilities to apply a family of possible worlds corresponding to a single model of modal logic, I take probabilities to apply to a family of models of *tense* logic.

probabilities given by quantum mechanics concern instantaneous projections.[34] My suggestion is that this indicates that the probabilities given by quantum mechanics concern not measurements by observers, but instead the determinate characteristics of events that occur independently of our intentions or decisions by means of indeterministic processes.

7. Times and Chance: Temporal Indeterminism

In the foundations of quantum mechanics literature, a 'no-go' result known as Pauli's Theorem has been taken to be problematic for the existence of so-called *time observables*. However, the idea that Pauli's Theorem rules out the use of the quantum state to assign probabilities to the time of decay is quite mistaken because it does not rule out the sort of assignment I proposed in Section 5. Instead, Pauli's Theorem shows that many existing interpretations of quantum mechanics cannot accommodate probabilities that refer to time intervals. So, rather than being being a problem for my account, I claim that Pauli's Theorem shows that the orthodox account—or any account based closely upon it, like quantum logic—cannot provide these probabilities, as I now demonstrate.

According to the orthodox account, probabilities in quantum theory are given by the Born Rule, which assigns a real number to the inner product of two state vectors. More generally, the quantum state assigns to any observable (i.e., self-adjoint operator) an *expectation value*. Projections are observables of a special kind: they take only values 0 and 1 on measurement (these are the eigenvalues of the observable), and the expectation value of a projection for a state (lying between 0 and 1) is interpreted as the probability of finding the result 1 on measurement, in which case the orthodox account says the corresponding proposition is true and that the system does indeed possess the property in question.

My alternative account can be seen as maintaining that there are certain observables which are not projections that can nonetheless be interpreted as giving the chances for the time of an event to lie within a time interval. I proposed that these chance propositions take the form of conditional probabilities, and so the corresponding observables lead to expectation values that lie between 0 and 1 (which is a necessary condition for their interpretation as probabilities). In the case considered above—the time of decay—we may define a family of observables $E_{[a,b]}$ (i.e., self-adjoint operators) whose expectation values in the state $|\psi\rangle$ correspond precisely to the probability $\Pr(a \leq t_d < b)$, as defined by previously by (3). (See the Appendix for details.)

The expectation values of these observables, then, provide an assignment of (conditional) probabilities to intervals of time. If these observables turned out to be projections then the orthodox account could interpret them as properties of the system in the usual way. However, they are *not* projections, and—to the extent that they are observables of the orthodox account—provably they cannot be projections. I now argue for that claim.

Generally speaking, it is not enough that such a probability assignment applies to time intervals, it must also properly respect the principle of *time translation covariance*. To explain: just as

[34]In contrast, the idea that the spatial position of an event is governed by an indeterministic process is not necessarily incompatible with the idea that quantum mechanics concerns instantaneous projections.

space is assumed to be homogeneous as far as the laws of physics are concerned, it is a fundamental assumption of physics that there is no absolute distinction between moments of time. That is, just as the results of an experiment are not affected by precisely *where* in space it takes place, neither is there a dependence on *when* an experiment takes place, absolutely speaking. If an otherwise identical experiment were to run in the morning or in the afternoon, all other things being equal, the results would be exactly the same.

To respect this principle in quantum physics, we must require of our probability distribution that it does not depend on the absolute value of the times themselves but only the relative temporal distance between them. We do this by demanding time translation *covariance* of the probability distribution. This means that, given the predictions for an experiment that begins at noon, we can generate the predictions for the same experiment beginning at two o'clock just by shifting the predicted results forward in time by two hours.[35] All the information about the results of the experiment itself is, therefore, contained in the relative temporal distances without making reference to absolute values of time.

Another standard (and even more general) assumption in quantum theory is known as the 'spectrum condition.' The spectrum condition concerns the allowed energies of a physically meaningful quantum system. It maintains that not only do we find that physical systems have a lower bound on their allowed energies (i.e., a value of energy below which the system cannot drop), but we would also like to rule out the idea that any such system is physically possible. Schrödinger (1931) seems to have been the first to consider such a possibility (in connection with the existence of an ideal quantum clock) and he concluded that a system with no lower bound on its energy is "physically meaningless" and "in contradiction with the foundations of quantum mechanics" (from Hilgevoord (2005)).

The inadequacy of the orthodox account of quantum mechanics with respect to probabilities for event times (such as the time of decay) can be seen in the inconsistency of the following three conditions:

1. There exist a set of (mutually orthogonal) projections whose expectation values provide a valid assignment of probabilities to time intervals.

2. That assignment of probabilities to time intervals covaries with time translations.

3. The energy of the system is bounded from below, i.e. there exists a value of energy lower than the energy of any possible state of the system.

These conditions are inconsistent because there is provably no covariant assignment of projections to intervals of time if the energy of the system is bounded from below (Halvorson, 2010; Srinivas & Vijayalakshmi, 1981). This result is known as Pauli's Theorem. There is no room for doing without conditions 2. and 3. However, a straightforward modification of 1. removes the inconsistency.

[35]Note that this is exactly what we did in Section 3.1 to update the probabilities for decay after conditionalization on non-occurrence.

The first condition says that the expectation values of the set of projections behave like a probability measure, in the sense of Kolmogorov. In the quantum mechanical parlance, we say that such a collection of observables forms a Projection-Valued Measure (or PVM). The existence of such a set of *projections* is ruled out by Pauli's Theorem. However, it turns out that requiring the observables to be projections is not necessary for the expectation values to behave like a probability measure. That is, by replacing 1. with 1′. we avoid contradiction:

1′. There exists a set of observables whose expectation values provide a valid assignment of probabilities to time intervals.

The observables I define (in the Appendix) to represent occurrence within a time interval form instead what is known as a Positive Operator Valued Measure (or POVM), whose expectation values also behave like a probability measure. In fact, every PVM is a POVM, and the only difference between them is that the observables of a POVM are positive operators that are not projections, i.e, a POVM whose observables are (orthogonal) projections is a PVM.[36] The inconsistency of the three original conditions entails that no time covariant POVM is a PVM, but there are plenty of time covariant POVMs (as I show in the Appendix).

This rules out the use of projections to assign probabilities to times, or to describe the time of an outcome. The physicists' folklore that Pauli's Theorem shows that time is different from space in quantum mechanics is, therefore, backed up that by the fact that there is no assignment of projections to describe location in time. This shows that the orthodox account, or any account that takes only projections to provide valid assignments of probabilities, cannot accommodate the resolution of Schrödinger's cat paradox I have argued for here. This also demonstrates that the value state of van Fraassen's modal account cannot determine the time of an event (at a world).

Perhaps more interestingly, this result also shows that the quantum mechanical account of the processes that lead to such events is indeterministic in a particularly robust way. In particular, the fact that no such assignment of projections to times exists means that there is no state that one could prepare (of *any* given quantum system) that would lead with certainty to the occurrence of an event within a given time interval.[37] This means that if I prepare the same state at time $t = 0$ on many occasions it must, of necessity, lead to many different durations before that that event occurs.[38] *Any* quantum mechanical process, that is, behaves like radioactive decay in this important respect.

[36]A projection is a positive operator, and positive operators are self-adjoint. A projection has the spectrum $\{0, 1\}$ and the positive operators, A, of a POVM have the spectrum $[0, 1]$. This means that they also satisfy the requirement that $\mathbb{O} \leq A \leq \mathbb{I}$, necessary for interpretation of an expectation value as a probability.

[37]More precisely, this claim arises as a corollary of a result from which Pauli's Theorem also follows. That result is known as Hegerfeldt's Lemma (Hegerfeldt, 1998). Assume for RAA that such a state, $|\psi_I\rangle$, may be prepared at time $t = 0$ which guarantees occurrence in some later (bounded) time interval I. Then there exists a projection P_I corresponding to occurrence in I whose range includes (at least) $|\psi_I\rangle$. Applying time translation covariance to P_I we have $P_{I+t} = U_{-t}PU_t$. Take a positive real number $\delta > |I|$, i.e. let δ be greater that the length of I. Then $P_{I+\delta}$ is a projection that corresponds to occurrence outside of I. By assumption, $|\psi_I\rangle$ has zero probability of occurrence during $I + \delta$ so that $f(\delta) = \langle\psi_I|P_{I+\delta}\psi_I\rangle = 0$, for all $\delta > |I|$. Note that δ may take any value within some open interval, i.e. the set of values of δ for which this equality holds has finite Lebesgue measure. By Hegerfeldt's Lemma, $\langle\psi_I|P_{I+\delta}\psi_I\rangle = 0$ for *all* δ, including $\delta < |I|$. The probability that $|\psi_I\rangle$ leads to occurrence during I is, therefore, zero, in contradiction with our original assumption. Thus there exists no such state $|\psi_I\rangle$ (RAA). For a proof of Pauli's Theorem along these lines, see Halvorson (2010).

[38]Since the predictions are probabilistic, 'of necessity' must be understood as holding in the long run.

This backs up the idea that quantum mechanics leads to *temporal indeterminism*, defined in Section 2 as follows:

Temporal Indeterminism There exists a set of physically possible worlds (i.e. sharing the same laws) each of which corresponds to a distinct, determinate time of decay, $0 < t_d < \infty$, such that any pair of these physically possible worlds match up to time $t = 0$ but diverge afterwards with respect to the time of decay, t_d.

To this I would add, moreover, that this applies to the time of *any* quantum event, not just radioactive decay, and that the actual world, of which quantum mechanics is true, appears in that set of worlds.

The specific way in which two worlds match at $t = 0$, I claim, is that they agree about the quantum state, which I also take to screen off the influence of past events. That is, let B be some event in the past and let A be some event in the future. Then, given that the correct quantum state in a world is $|\psi\rangle$ at $t = 0$, I take it that the occurrence of B makes no difference to the chance of A at $t = 0$, that is, $\Pr(A|B \,\&\, |\psi\rangle) = \Pr(A|\, |\psi\rangle)$.

So this is just what it means for the same laws to hold at the worlds w_X: the quantum state gives the chances in each world by screening off past events. For the state to be complete at $t = 0$, I suggest the following conditional: the state of a system is complete *iff* it gives the correct chances. This makes the completeness of the state a time dependent matter since the set of worlds described by the chance propositions (and correspondingly the quantum state) changes with time. That is, not all worlds w_X will be assigned the same chances at $t_o > 0$ because the chances depend on the admissible evidence, which changes over time.

In particular, recalling the discussion of Section 3.1, the phenomenological decay law led to the same assignment of probabilities to the event of a particular nucleus undergoing radioactive decay during the *next* given time interval, given that it had not yet decayed. For that process to be described in the same way by quantum mechanics, it must be the case that neither (i) is the time of decay determined by the state; nor (ii) are the probabilities of decay during a given time interval fixed by the probabilities at an earlier time.

These characteristics are particularly hard to square with an account that takes the quantum state to represent the determinate state of some physical object, rather than as providing an encapsulation of the chances for the occurrence of future events. My contention is that this difficulty lies at the heart of the paradoxes surrounding measurement in quantum mechanics quite generally, and that resolving the conflict in favor of the latter idea provides a resolution of those paradoxes, as well as Schrödinger's cat paradox. I now argue for that claim in a particular case.

8. Resolving Wigner's Friend Paradox

The notion of temporal indeterminism, coupled with my general proposal for providing quantum probabilities for the time of an event, provides a novel resolution of not just Schrödinger's cat paradox but another notorious paradox of measurement constructed along the same lines: Wigner's friend paradox. Wigner (1983) considers an entirely prosaic example of a measurement situation that nonetheless leads him to a quite fantastic conclusion: that consciousness has a special role to play in the interpretation of the theory, viz. it is contact with a *conscious* observer that collapses the wavefunction. Here is the situation Wigner considered.

Wigner waits outside the laboratory, perhaps enjoying a cigarette, while his friend and lab partner enters and checks the results of a measurement that they had left running before their break. The friend diligently records the result in his log book and updates the state of the atomic system accordingly. According to him, the atomic system is no longer in a superposition with respect to the measured observable. Wigner, however, waiting in isolation outside continues to update the state of the system according to the usual unitary evolution given by the Schrödinger equation. Knowing that his friend has entered the lab, Wigner also describes how the resulting quantum mechanical correlation will lead to a superposed state of friend, apparatus and system.

Yet Wigner also knows that when his friend returns outside to report on the results he will maintain that he witnessed a definite outcome. The same outcome, moreover, that Wigner would witness were he to go inside and check. The two accounts of the situation—one given by Wigner, the other by his friend—are in contradiction, but both are in accord with the orthodox account of quantum measurement. Who is correct? Wigner reasons, by means of a principle of charity, that his friend was right and he was wrong. After all, Wigner knows that he can check up on his friend and if he does quantum mechanics guarantees he will find the same result. What explains the disparity? Well, Wigner concludes, his friend possesses something to which neither the system nor measurement apparatus can lay claim: conscious awareness. This *consciousness*, therefore, must be the cause of collapse.

This is an unacceptable solution. By all means, let quantum mechanics play a role in our understanding of consciousness (although skepticism on that front would be wise), but let's not rest our interpretation of quantum mechanics on our (very limited) understanding of consciousness. So what are the alternatives? The logic of this situation appears to dictate the possible answers: (i) we may claim neither description is correct; (ii) we may claim that the descriptions are in conflict but both are correct; (iii) we may claim that only one is correct. Confirming the old adage that philosophical opinions are like gases (in that they expand to fill the available logical space), we find each option with advocates in the literature.

Like Wigner, the Everettian takes option (iii) but sides with Wigner against his friend. According to Everett, that is, the experience as of a definite result should not be taken as evidence that other alternative outcomes did not obtain. Relational accounts aim to pursue option (ii), making some radical claims about the perspectival nature of reality in the process (Dieks, 2009; Rovelli, 1996). I will not consider these further here.

In contrast, informational interpretations of the quantum state grab (i) by the horns and deny that the quantum state is apt to represent an objective reality at all. As Timpson puts it:

> according to the information view, there is just not an issue here: one is being misled by a jejune literalism about the quantum state. There is no mysterious collapse coming into

> play at some point or other; nor is there any troublesome hanging-in-limbo for the poor old friend. Rather, both agents involved (Wigner, and friend) simply ascribe different states to the system being measured, without contradiction. There is not supposed to be one correct state which is in some sense an objective property of a system; rather, each agent will ascribe a different state based on their differing information (Timpson, 2013, pp. 148–149)

But, Timpson argues, the intuition that underlies this idea—that both observers simply represent the system as best they can with the information they have—conflicts with the core of the everyday concept of information, which is at root an *epistemic* notion. Information, claims Timpson, is factive. That is, if one can be truly said to have the *information that* p, then one *knows that* p. And one cannot know p unless p is in fact the case. Talk of information, then, just serves to obscure the issue, which still concerns whether Wigner or his friend may lay claim to knowledge of the system, and about *that* at most one of them can be right.

Here I agree with Timpson: where they disagree, at most one description can be correct, and who is correct depends on how things are with the system. However, I want to reject the terms of the proposed solutions by maintaining that *both* Wigner and his friend are right (at first), and *then* both are wrong—and, crucially, that their descriptions become wrong *before* the friend checks the measurement result. My solution of the paradox follows from the assumptions that (a) an experiment has an outcome at some definite time, and (b) that the quantum state supplies the chances for the occurrence of that event during some time interval.

When one of the observers is wrong about whether or not an outcome has obtained, then, it is because they are wrong about the chances. In particular, when the outcome has obtained that fact becomes admissible evidence and, therefore, for their credences to reflect the chances, Wigner and his friend must both update their credences *before either one of them has the evidence to do so*. Whether or not they are correct about the system at time t doesn't depend only on what they believe about the system or the information they have, it also depends on what they don't yet know: whether or not, as of t, the experiment has an outcome.

Although Wigner's friend is in a position to update his credences so that they match the chances before Wigner, who waits outside the lab, the change in the friend's epistemic position does not affect the chances (which changed the moment an outcome obtained). That is, after the outcome has occurred, both observers (assigning their credences in ignorance of that fact) are equally incorrect about the chances. Before the outcome obtained, however, they were both equally correct. What makes Wigner's description of the system—*and* his friend's—right or wrong is an objective fact that obtains independently of the friend's activities or beliefs. Let's go through this in a little more detail.

As with the decay of the atom in Section 2, we represent the proposition that the detector registers a result with the present tense φ. Take, for example, the appearance of a lone dot on a photoluminescent screen. Assume that the dot appears at time t_d at a world w. Then φ is true at time $t = t_d$. According to our tensed semantics, $\mathsf{P}\varphi$ is true at a time t (at a world) if $t_d < t$ and false if $t < t_d$. If $\mathsf{P}\varphi$ is still false, Wigner's superposed state is the right one to assign. If, however, $\mathsf{P}\varphi$ is true then a definite result has obtained and Wigner's assignment is wrong. Wigner's friend is in exactly the same boat: his 'observation of the system' makes absolutely no difference to the truth conditions of the crucial proposition $\mathsf{P}\varphi$.

So, as in the case of Schrödinger's cat, we are able to conclude that at each time there is a matter of fact as to whether or not an experimental outcome has obtained, and hence the time at which an outcome obtains determines who is right and who is wrong in matters of the quantum state—which, remember, means giving the correct chances regarding the time of the outcome.

What makes this resolution of the paradox possible is the idea that Wigner and his friend can use the quantum state to give chances for the time of occurrence by conditioning on the fact that an outcome obtains (as the orthodox account assumes) but without committing themselves to a particular time at which it obtains. Since the chances change with time as the admissible evidence changes, this explains the illusion of subjectivity in this case. In fact, as soon as the experiment has registered an outcome the correct chances (and thus the correct state) change independently of anyone's beliefs or actions—Wigner's friend has nothing to do with it.

9. Conclusion

In attempting to sum up the approach to interpreting quantum theory advocated here, I was reminded of the following well-known remark by E. T. Jaynes:

> But our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature — all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple. (Jaynes, 1990).

I have argued above that quantum mechanical probabilities, conceived as Lewisian chances, can apply to propositions about the time that an outcome obtains. The truth or falsity of such propositions does not depend on the quantum state but rather objective facts about the course of events of the world. The dependence of the objective chances on the admissible evidence, and thereby on *time*, explains the apparent dependence of the state on the actions and beliefs of the observer. This subjective aspect is, however, ultimately an illusion: outcomes obtain independently of the observer and at determinate times. This insight provides a vital component of my proposal for unscrambling the quantum formalism, which I call the Stochastic Histories interpretation.

In concocting the original scrambled mess (i.e., the so-called Copenhagen interpretation) Heisenberg deserves especial blame, and if we can reach an understanding of the theory that allows us to avoid the pull of the following thought (expressed by Heisenberg at the Solvay conference of 1927) then we will have made real progress:

> I do not agree [... that] nature makes a choice. [...] I should rather say [...] that the observer himself makes the choice, because it is only at the moment when the observation is made that the 'choice' has become a physical reality [...] (Bacciagaluppi & Valentini, 2009, pp. 449–450)

Indeed, it seems to me that the reason why the resolution of the paradoxes surrounding measurement that I have articulated here has not been seriously considered is that it has been part of the folklore of quantum mechanics from the very beginning that the probabilities it supplies concern measurements performed by an observer at a time of their choice.

This means that not only does the observer get to specify which *kind* of events are to be assigned probabilities by the quantum state (through her choice of which apparatus to use), but also that the very instant at which those events are supposed to occur is a matter of her free choice. This idea is not without theoretical justification: quantum mechanics most readily supplies probabilities for instantaneous Heisenberg picture observables that do indeed represent a family of instantaneous events. But the idea that in actuality an experimenter may *choose* the time of an experimental outcome is patently absurd.

Just as you cannot bring about the decay of a radioactive atom at a moment of your choosing by a sheer act of will, neither can you summon a luminescent dot to appear at the precise instant you would like, nor expect that a cosmic ray will enter your cloud chamber on demand. But this idea suffuses the mainstream interpretation of the propositions of quantum logic as 'experimental questions' to which the system is compelled to answer 'yes' or 'no' (due to Mackey).[39] These propositions, corresponding to projections, are necessarily instantaneous (Pashby, 2014a). The idea that instantaneous propositions correspond to *questions* leads to the seemingly inescapable conclusion that, although she may not control *which* outcome obtains, the experimenter may decide *when* an outcome obtains.

The supposition that the experimental physicist possesses a preternatural ability to bring about the existence of truth values at her whim lay at the heart of Schrödinger's cat paradox, and Wigner's friend paradox. This mistaken idea also infects foundational discussions of quantum mechanics in seemingly unrelated areas. Take, for example, the claim of Malament (1996) that in choosing the instant of her detection experiment the experimenter may bring about an illicit *act-outcome correlation* at spacelike separation. His argument that this constitutes an unacceptable violation of relativistic locality rests on the claim that an observer may choose not just where to set up her detector but also precisely *when* the experiment occurs—without that assumption the observation of an instantaneous outcome could not be construed as an *act*.

The attempt to avoid controversial matters of interpretation by retreating to Mackey's operationalist account of the theory is understandable. But in deflecting attention to the role of the experimenter we risk reducing the experimental physicist to the questioner in a parlor game of 'twenty questions,' while simultaneously attributing to her supernatural powers of affective inquisition. Operationalism is itself a philosophical position, and in attempting avoiding assumptions

[39]For a philosophical account along these lines, see Stein (1972).

about the nature of physical systems we were led to attribute a far too implausible character to the nature of the experimenter. Let us accept our limitations: in performing a typical experiment we set up the apparatus, we prepare the system, and we wait for the outcome. No other account is justified by the practice of experimental physics.

Another source of confusion on this point arises from the supposed prohibition of so-called *time observables* in quantum theory. According to folklore, time is not an observable of the theory and so the theory does not have the means to provide probabilities for times. However, the latter conclusion does not follow: there is no prohibition on using the quantum mechanical state to provide a probability distribution for the *time of an event*, as I have shown here. (See the Appendix for more details.) If we allow that the quantum state supplies the chances for the time of an outcome then we have the same quantum mechanical account of *when* an outcome obtains as we do of *which* outcome obtains. We should not expect more of a probabilistic theory. We should, however, expect at least that much from an interpretation of quantum mechanics.

Appendix: Event Time Observables in Quantum Mechanics

According to quantum logic, the set of propositions that are truth-apt corresponds to the lattice of projections of a quantum system. This defines a non-Boolean algebra which admits of certain Boolean subalgebras concerning (e.g.) all the possible momental (or positional) properties of the system. Given some definite quantum state, the Born Rule assigns a real number between zero and one to any proposition corresponding to a projection. This is known as the *expectation value* for the projection in that state.

The restriction of an assignment of expectation values by the state to just the lattice of projections is justified by Gleason's Theorem, which states (roughly) that an assignment of probabilities to the lattice of projections uniquely determines a quantum state, and vice versa. That is just to show, however, that a restricted assignment of expectation values to the lattice of projections suffices to determine a quantum state. This makes the interpretation of the state according to quantum logic *possible*; it does not make it necessary, nor (as I now argue) adequate.

In the most general setting, a quantum state determines an assignment of expectation values to the *algebra of observables*, and vice versa.[40] The lattice of projections (or rather the non-Boolean algebra so defined) forms a subalgebra of the algebra of observables. The elements of this subalgebra are just the observables whose expectation values (in a given state) may be interpreted as probabilities for the possession of properties. If the probability of possession is one then the system is said to possess that property and the corresponding proposition is true.

The lattice of projections does not, however, provide the maximal subalgebra of observables with expectation values that may be interpreted as probabilities *simpliciter*, i.e., with no reference to properties or their possession. The maximal subalgebra of observables whose expectation val-

[40]In the algebraic approach to quantum theory, an algebraic state is an assignment of expectation values to every element of the algebra of bounded operators, $\mathcal{B}(\mathcal{H})$ (a von Neumann algebra), and every *normal* algebraic state corresponds to a density operator on $\mathcal{H}$, i.e. a standard quantum state.

ues are appropriate for interpretation as probabilities is known as the *algebra of effects*, and it too contains the lattice of projections as a subalgebra. The operational account of quantum theory maintains that only the full algebra of effects contains the necessary resources for predicting the correct probabilities concerning the results of actual experiments. As was shown in Section 7, there are indeed certain observables that must lie outside the lattice of projections (but within the algebra of effects) that are empirically meaningful: observables giving the chances for the time of an event.

I will show here how my proposal for these observables fits with two fundamental principles of the theory: (i) that the state of a system corresponds to a density operator (a positive trace class operator with trace one); (ii) that conditional probabilities are given by Lüders' Rule. Recall that Gleason's Theorem shows that (so long as the state space is a separable Hilbert space of dimension ≥ 2) a countably additive probability measure defined on the propositions about the system (i.e. the lattice of projections) uniquely determines a density operator, and vice versa. However, Busch (2003) shows that an equivalent result holds for the algebra of effects, in which case any observable lying within the algebra of effects is just as legitimate for interpretation as a probability as a projection.[41]

Cassinelli & Zanghi (1983) show that if the state is given by a density operator then Lüders' Rule is the only possible expression for a conditional probability. I now show how my proposal fits with this interpretative rule.[42] The expression (3) I gave in Section 4 for the probability of decay during an interval of time defines an operator $E_{a,b}$ on a Hilbert space $\mathcal{H}$ as follows:

$$\langle \psi | E_{[a,b]} \phi \rangle := \int_a^b \frac{\langle \psi | P_\varphi(t) \phi \rangle}{\int_0^\infty \langle \psi | P_\varphi(t) \phi \rangle dt} dt,$$

for all $\psi, \phi \in \mathcal{H}$ (or at least for some non-trivial domain). So defined, the operator $E_{[a,b]}$ is a bounded positive operator, and thus its domain may be extended to the entire Hilbert space.

A density operator W assigns to every bounded self-adjoint operator $B \in \mathcal{B}(\mathcal{H})$ an expectation value through the *trace prescription*:

$$\langle B \rangle_W = Tr[BW].$$

In Section 4, I proposed to interpret the expectation value $\langle E_{[a,b]} \rangle_W$ as the chance at t of decay during $[a, b]$ for an unstable atom in state W. This interpretation can easily be extended to other similar operators, defined by another family of Heisenberg projectors $P_\varphi(t)$. But let us now consider why $\langle E_{[a,b]} \rangle_W$ is apt to play the role of chance.

Mathematically, we represent the possible experimental outcomes of some experiment by means of a measurable space $(\Sigma, \mathcal{X})$, where $\mathcal{X}$ is a σ-algebra defined on Σ, a nonempty set. An assignment of probabilities to each outcome $X \in \mathcal{X}$ is given by a map $p : \mathcal{X} \to \mathbb{R}$. This leads to the following definition.

[41]See Ch. 3 of Pashby (2014c) for further argument to the effect that the algebra of effects is in many ways a more natural companion for the quantum state than the lattice of projections.

[42]For further discussion see (Pashby, 2014c).

Definition 1. *The triple* $(\Sigma, \mathcal{X}, p)$ *is a probability space and* p *is a (generalized) probability measure if the following conditions are met:*

1. $p(X) \geq 0$ *for all* $X \in \mathcal{X}$ *(positivity)*
2. $p(\Sigma) = 1$ *(unity)*
3. $p(\cup_i X_i) = \sum_i p(X_i)$ *for countable, mutually disjoint families* X_i *(σ-additivity)*

In the case considered, we are concerned with outcomes lying within time intervals $[a, b]$, with $a < b$ and $0 < a$, and thus the Borel subsets of the positive real line, $\mathfrak{B}(\mathbb{R}^+)$, provide an appropriate measurable space.

To ensure that p_W is a valid probability measure, we need to lay down some conditions that the assignment of operators $X \mapsto E(X)$ must satisfy. First, since W has unit trace, we require that $E(\Sigma) = \mathbb{I}$, which ensures that $\Pr(\Sigma) = 1$. Similarly, to ensure that $\Pr(X) \geq 0$ we require that $E(X)$ is a positive operator, $E(X) \geq O$.[43] The final condition we need to ensure is $\sigma-$additivity, which is achieved as below, providing the definition of a (normalized) Positive Operator Valued Measure (POVM).

Definition 2. *A normalized Positive Operator Valued Measure (POVM)* E *is a map from a measurable space* $(\Sigma, \mathcal{X})$ *to the bounded operators on a Hilbert space* $E : \mathcal{X} \to \mathcal{B}(\mathcal{H})$ *such that*

1. $E(X) \geq O$ *for all* $X \in \mathcal{X}$ *(positivity)*
2. $E(\Sigma) = \mathbb{I}$ *(unity)*
3. $E(\bigcup_i X_i) = \sum_i E(X_i)$ *(in the weak operator topology) for countable, mutually disjoint families of* X_i *(weak* $\sigma-$*additivity)*

The requirements suffice to determine that $E(X)$ is a self-adjoint operator with spectrum $[0, 1]$ since, therefore, $\mathbb{I} \geq E(X) \geq O$. So long as the $E_{[a,b]}$, defined above, form the elements of a POVM, then, the trace prescription provides a probability measure.[44]

But in what sense is $\langle E_{[a,b]} \rangle_W$ a *conditional* probability? Just as a density operator defines an expectation value for every bounded operator through the trace prescription, so too does a countably additive valuation $\omega(B)$ of every bounded operator B define a (not necessarily normalized) density operator. W.[45] The following valuation defines a density operator W_E in the same manner

$$\omega_E(B) := \frac{\omega(EBE)}{\omega(E)} \qquad \text{for all } B \in \mathfrak{B}(\mathcal{H}).$$

[43] An operator A is positive, $A \geq O$ if $\langle \psi | A\psi \rangle \geq 0$ for all $\psi \in \mathcal{H}$. If bounded, A is therefore self-adjoint. This defines a partial order on (bounded) self-adjoint operators A, B, i.e. $A \geq B$ if and only if $A - B \geq O$.

[44] See Brunetti & Fredenhagen (2002) for further examples of event time observables such as this.

[45] That is, $\omega(B)$ defines a *normal* state on the algebra of bounded operators $\mathfrak{B}(\mathcal{H})$ which determines a unique (though not necessarily trace one) density operator if $\dim(\mathcal{H}) \geq 2$.

Now, if E is a projection (i.e. $E^2 = E$) then $\omega_E(B)$ corresponds to a properly normalized density operator W_E which may be regarded as the state of the system conditioned on the truth of the proposition E. In that case, if $F \leq E$ is another projection (where $\leq$ is the relation of subspace inclusion) then

$$\omega_E(F) = \frac{\omega(EFE)}{\omega(E)} = \frac{Tr[EFEW]}{Tr[EW]} = Tr[FW_E]$$

is the probability of F in the state W conditioned on the truth of E. This is Lüders' Rule for conditional probabilities, and it is easily seen that $\omega_E(E) = 1$ as it should.

To see how $\langle E_{[a,b]} \rangle_W$ fits this mold, consider the family of operators defined (on an appropriate domain) as follows:

$$\langle \psi | F_{[a,b]} \phi \rangle := \int_a^b \langle \psi | P_\varphi(t) \phi \rangle dt.$$

We may now write

$$E_{[a,b]} = \frac{F_{[a,b]}}{F_{[0,\infty)}}.$$

Consider the state valuation defined by:

$$\omega_{F_{[0,\infty)}}(B) = \frac{\omega\left(F_{[0,\infty)}^{1/2} B F_{[0,\infty)}^{1/2}\right)}{\omega(F_{[0,\infty)})},$$

where $F_{[0,\infty)}^{1/2}$ is the unique positive operator such that $F_{[0,\infty)}^{1/2} F_{[0,\infty)}^{1/2} = F_{[0,\infty)}$. This defines an unnormalized density operator $W_{F_{[0,\infty)}}$, and this slight generalization of Lüders' Rule is known as a *Lüders operation*.

The expectation value of $E_{[a,b]}$ for the state W conditioned on $F_{[0,\infty)}$ may thus be written as[46]

$$\begin{aligned}
\langle E_{[a,b]} \rangle = \frac{Tr[F_{[a,b]}W]}{Tr[F_{[0,\infty)}W]} &= \frac{Tr[F_{[0,\infty)}^{1/2} F_{[0,\infty)}^{-1/2} F_{[a,b]} F_{[0,\infty)}^{-1/2} F_{[0,\infty)}^{1/2} W]}{Tr[F_{[0,\infty)}W]} \\
&= \frac{Tr[F_{[0,\infty)}^{1/2} D_{[a,b]} F_{[0,\infty)}^{1/2} W]}{Tr[F_{[0,\infty)}W]} \\
&= Tr[D_{[a,b]} W_{F_{[0,\infty)}}],
\end{aligned}$$

where $D_{[a,b]} = F_{[0,\infty)}^{-1/2} F_{[a,b]} F_{[0,\infty)}^{-1/2}$, known as the *operator normalization* of $F_{[a,b]}$.

This above expression, I submit, deserves its interpretation as the conditional probability of decay during $[a, b]$ given decay at some time $0 \leq t_d < \infty$. Note that it may be interpreted here as a properly normalized *conditional* probability since we have

$$\langle E_{[0,\infty)} \rangle_W = \omega_{F_{[0,\infty)}}(D_{[0,\infty)}) = \frac{Tr[F_{[0,\infty)}W]}{Tr[F_{[0,\infty)}W]} = 1.$$

[46]Note that $F_{[0,\infty)}^{-1/2} F_{[0,\infty)}^{1/2} = F_{[0,\infty)}^{1/2} F_{[0,\infty)}^{-1/2} = \mathbb{I}$.

The only departure required from Lüders' Rule (narrowly defined) is the use of a positive operator $F_{[0,\infty)}$ to condition the state rather than a projection, and the resulting use of the operator normalized POVM $D_{[a,b]}$ and the unnormalized $W_{F_{[0,\infty)}}$ in order to fit the trace prescription.

If, however, conditionalization by means of a genuine projection is desired then one needs only to consider projections on the Hilbert space formed according to the continuous direct sum $\mathfrak{H} = \bigoplus \mathcal{H}_t dt$. On this 'temporally extended' Hilbert space, time intervals may be mapped to genuine projections, and we can use Lüders' Rule proper to return the very same expectation value $\langle E_{[a,b]} \rangle_W$. For further details, see Ch. 8 of Pashby (2014c).

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