

1. How to tell if an argument is valid.

If an argument is an instance of a **valid argument form** then it is a valid argument. What is a valid argument form, you ask? Some examples:

- (a) 1. All As are Bs.
 2. This is an A.

 ∴ This is a B.

(Categorical Syllogism)

- (b) 1. If A then B.
 2. A

 ∴ B

(Modus Ponens or Material Implication)

- (c) 1. If A then B.
 2. Not B

 ∴ Not A

(Modus Tollens)

- (d) 1. Either A or B.
 2. Not B

 ∴ A

(Disjunctive Syllogism)

Filling in word phrases for the letters A and B to make declarative sentences, you can use these valid argument **forms** to generate any number of valid **arguments** that you can use to impress your friends!

This is not an exhaustive list but the idea is that a valid argument is one that, when the premises are true, **guarantees** the truth of the conclusion, just from the form of the argument. In this precise sense, a valid argument form is truth **preserving**: if you give it true premises, it gives you back a true conclusion. Always, with **no exceptions**.

Therefore, a valid argument has no **counterexamples**. If you can find a counterexample, then the argument cannot be valid—it is **invalid**.

What is a counterexample, you ask?

A counterexample is a way that the premises could be true while the conclusion is false, without being inconsistent. (Inconsistency arises, remember, when the premises and the conclusion can't all be true at once, i.e., when they are in

contradiction. Example: It would be inconsistent to believe both A and not A; they can't both be true together—this is known as the Law of Excluded Middle. If the argument is valid then it won't be possible to deny the conclusion and affirm the premises without inconsistency. This is another way of saying that a valid argument is one that guarantees the truth of the conclusion given true premises.)

A counterexample shows how the truth of the premises fails to guarantee the truth of the conclusion. If you can find a single counterexample then the argument is invalid. You can think of the counterexample as describing a situation in which the premises are true and showing how in that situation something other than the conclusion would be true and, moreover, that the conclusion would be false.

Finding a counterexample to an argument doesn't make the conclusion false, it just shows that the argument is invalid. The most tempting logical fallacies (=invalid arguments) are those where the conclusion seems plausible but the premises are not sufficient to get you there. However, remember that a valid argument can have a false conclusion. (How? By having one or more false premises, of course!) A counterexample works by displaying the logical gap between what the premises allow you to deductively infer and what the truth of the conclusion requires.

2. Science and Logic

Some invalid arguments are used by scientists in practical reasoning to good effect. (But occasionally they will get things wrong—such is the risk with an invalid argument.) Some of these arguments seem so reasonable, or so unavoidable (in the sense that science would be impossible without such reasoning) that they get special names and philosophers of science devote lots of effort to trying to explain to other philosophers why scientists are justified in reasoning this way, despite the fact that scientists using these ways of reasoning commit logical fallacies.

We have encountered two such forms of reasoning that are deductively invalid, but useful nonetheless (and possibly unavoidable in doing science). One way to describe these forms of reasoning is to say that they are **ampliative**. That is, you get out more than you put in (in the sense that the truth of the premises doesn't quite guarantee the truth of the conclusion). That sounds great, but in reasoning this way you take on a lot more risk than using a valid argument form. (Think here of putting money into a risky stock market investment rather than a savings account.)

(i) Induction

1. Every A I have ever seen is a B. _____
- ∴ All As are Bs.

It is easy to generate a counterexample: just imagine an A that you have not seen that is not a B.

Why is this way of reasoning useful in science? We can only make so many observations or do so many experiments. If I want to believe that my scientific theory fits the data in every possible case then I need to use induction. This applies to even the banal statement that every electron has the same charge. To confirm the truth of the statement I would have to find, and measure the charge of, every single electron in the universe that has ever existed. But scientists do (rightly) believe that every electron has the same charge! If I want to justify this belief, it can only be through inductive reasoning.

(ii) **Abduction**

1. This theory agrees with the data in every (possible) case.

∴ This theory is true, and what it says of the world is true.

You can also see that this argument is invalid. One way to generate a counterexample is to think of another theory (theory₂) that agrees with the data just as well as this theory (theory₁) but tells us something radically different about the world. If theory₂ is true then theory₁ can't possibly be true. But in this situation (where theory₂ is true) it is still true that theory₁ agrees with the data in every case. Therefore, the conclusion is false while the premise is true. That means we have a counterexample.

We have already met this situation in a historical context: the Copernican system and the Tycho system make exactly the same astronomical predictions but it seems that only one of them can be true since either the Earth moves or it doesn't (the Law of Excluded Middle again.) (Some philosophers of science claim that in practice this "underdetermination of theory by the evidence" is always resolved one way or the other by future experiments. This doesn't affect the claim that abduction is deductively invalid.)

If you are having trouble with the difference between abduction and induction, think about the theory that says the world is made up of atoms (the modern theory, that is, which says that atoms are composed of electrons and nuclei). Is there a difference between saying the theory is true and saying the theory gives the correct data in every case?

Yes! If the theory is true then **there are atoms**, and the behavior of these atoms explains what we see around us. If the theory just gives the correct data in every case (we say then that it is **empirically adequate**) then we cannot deduce from that fact that atoms exist, i.e. we can give no valid argument whose conclusion is that atoms really exist. If that were the end of the story then it would be better to read talk of atoms as a convenient fiction. (Atoms would be no more real than Sherlock Holmes.)

But do scientists go around talking as if atoms are fictional? No! This is because they reason by **abduction**, and so infer the truth of the theory from its success. If the theory is true (and atoms exist) then we get good explanations of so many things.

To license those explanations we need abduction (also called **inference to the best explanation**).

3. On Proof. We are yet to mention **proof**. What is a proof?

A proof is just a sound argument, i.e. a valid argument with true premises, although the valid argument form may be quite elaborate compared to our earlier simple examples. The idea here is that a proof of some fact establishes beyond all doubt that some conclusion (i.e. the fact) is true, and the best way to establish the truth of a conclusion is by means of a valid argument with true premises. Mathematical proofs are good examples of this, although they may in practice use some more elaborate argument forms than we have seen so far. (Note that mathematical facts are often true by definition, e.g., 1. A square has four sides of equal length. Therefore, every square has four sides of equal length.)

One proof method we have met is **indirect proof** (or reduction ad absurdum, or RAA). This is an instance of the following valid argument form, which is used here to establish the truth of A.

(e)	1. Not A	(Assume for indirect proof of A)
	2. If not A then B	(Premise)
	3. B	(From 1. and 2.)
	4. Not B	(Premise)
	5. B and not B	(Contradiction!)
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	\therefore A	(By Indirect Proof)

Why is this a valid argument form? Well, if premises 2. through 4. are true then the conclusion must be true. You can see from the following valid argument form that indirect proof proceeds by a valid argument, and is thus a valid argument form.

1. Either A or not A.	(Law of Excluded Middle)
2. If not A then both B and not B.	(From indirect proof)
3. It is false that both B and not B.	(Law of Excluded Middle)
4. It is false that not A.	(Modus Tollens)
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\therefore A	(From 1. and 4.)

So indirect proof relies on the Law of Excluded Middle (which is a logical truth) and Modus Tollens (which is a valid argument form). Therefore it is a valid argument form. (Phew! What a relief.)

(Another valid argument form often used in mathematical proofs that we have not encountered is **mathematical induction**. This is distinct from scientific induction and is a little more involved—and valid—but it won't be used in this course.)